Geometric and General Topology
Comprehensive Exam January 2009

Partial credit will be given for your work. Credit equivalent to five complete problems will guarantee a pass.

Geometric topology

1. Let X be the subset of $\mathbb{R}^3$ consisting of the unit sphere, the segment of the z-axis inside the unit sphere, and the unit disc in the xy-plane. Find the fundamental group of X.

2. Let $F : S^1 \to S^2$ be a continuous map.

   (a) Show that f is homotopic to a map that is not onto.

   (b) Show that f is homotopic to a constant map.

3. Let T be a punctured torus, i.e. a compact surface with boundary consisting of a torus with a small open disc removed. Find all connected two-fold covers of T, up to covering space isomorphism, and justify your answer. Let $f : S^1 \to \partial T$ be a homeomorphism from a circle to the boundary of the surface T. For which of the covers $\widetilde{T}$, if any, is there a lift $\widetilde{f} : S^1 \to \widetilde{T}$?

General topology

4. Show that every retract A of a Hausdorff space X is closed in X. Hint: Show that $X \setminus A$ is open.

5. Show that the product of two compact spaces is compact.

6. State the Urysohn Lemma, and show that every connected normal space with at least two points has uncountably many points.