Comprehensive Exam
General and Geometric Topology
(August 2008)

NOTE: Partial credit will be given for your work. Having accumulative credit equivalent to 5 complete problems will be sufficient to pass this exam.

(1) Give the definitions for a space to be path-connected and for a space to be locally path-connected.

Show that an open, connected subspace of a locally path-connected space is path-connected.

Give an example (with some rationale to support your claim; however, a precise proof is not required) of a connected space that is <u>NOT</u> path-connected.

(2) Find the fundamental group of the space produced by attaching a disc to a torus along an essential simple closed curve; i.e., by gluing the boundary of a disc to an essential simple closed curve in the torus by a homeomorphism.

(3) Let $\{C_n\}$ be a collection of closed non-empty subsets of a compact space X such that $C_{n+1} \subseteq C_n$, for all n. Use the definition of compactness to show that $\cap C_n \neq \emptyset$.

(4) Find all connected 3-fold covering spaces of the wedge of a circle and a projective plane (the **wedge** of a circle and a projective plane is the union of a circle and a projective plane where the two spaces have precisely one point in common).

(5) Let M be the surface obtained by identifying the edges of an octagon using the pattern $abcbdc^{-1}da^{-1}$. Compute the Euler characteristic of M, and identify which surface it is.

(6) Let $A \subset \mathbb{R}^2$ be a one-one (injective), continuous image of the closed unit interval. Show that A is a retract of $\mathbb{R}^2$.

Recall that the space Y is said to be a **retract** of the space X if $Y \subset X$ and there is a continuous function $r : X \to Y$, so that $r \circ i_Y = id_Y$, where i_Y is the inclusion map of Y into X and id_Y is the identity map of Y.