Geometric and Algebraic Topology

Comprehensive Exam – June 2017

Define your terminology and explain your notation. If you require a standard result, then state it before you use it; otherwise give clear and complete proofs of your claims.

Partial credit will be given for your work. Credit equivalent to five complete problems will guarantee a pass.

Geometric Topology

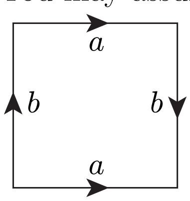
1. Let M be the Möbius strip, formed by gluing opposite sides of a rectangle to each other with a half twist. Let A be the annulus, formed by gluing opposite sides of a rectangle to each other without a twist. Show that M and A are of the same homotopy type. You may either give formulae, or describe the required maps geometrically.

2. Let K be the Klein bottle, with marked curves a and b, as shown in the figure below. By using an appropriate theorem, calculate the fundamental groups of the spaces obtained by attaching a disk to K along:

   (a) the curve a.

   (b) the curve b.

   You may assume knowledge of the fundamental group of the Klein bottle and its generators.



3. List the three-sheeted covering spaces (up to covering space isomorphism) of the torus $S^1 \times S^1$. Justify your answer.

Algebraic Topology (Unless indicated otherwise all homology and cohomology groups have integer coefficients.)

4. M is a closed, connected, orientable 3-manifold. Prove that the Euler characteristic $\chi(M) = 0$.

5. X is a space with $H_0(X) = \mathbb{Z}$, $H_1(X) = \mathbb{Z} \oplus \mathbb{Z}_{15}$, $H_2(X) = \mathbb{Z}_{10}$, and $H_p(X) = 0$ for all $p \geq 3$. Find $H_p(X; \mathbb{Z}_6)$ for all $p \geq 0$.

6. Let $X = S^1 \times S^1$. Let A be a simple closed curve in X which bounds a disk D in X. Use the long exact sequence of the pair (X, A) to compute $H_p(X, A)$ for all $p \geq 0$.