Geometric and Algebraic Topology
Comprehensive Exam – January 2016

Define your terminology and explain your notation. If you require a standard result, then state it before you use it; otherwise give clear and complete proofs of your claims.

Partial credit will be given for your work. Credit equivalent to five complete problems will guarantee a pass.

Geometric Topology

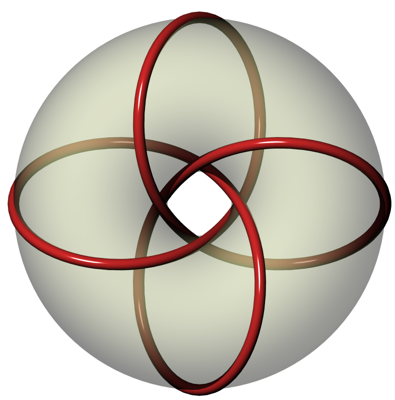
1. Let S be a closed orientable surface.

   (a) First, suppose that S has a triangulation with the property that the degree of each of the vertices is five. Show that S must be the sphere.

   (b) Now suppose that S has a triangulation with the property that the average (more specifically, the mean) degree of the vertices is five. Again, show that S must be the sphere.

2. State and prove the Brouwer fixed point theorem for the unit disk $D^2 \subset \mathbb{R}^2$.

3. Let $T \cong S^1 \times S^1$ be a torus embedded in S^3 in the standard fashion. The (p, q) torus knot is the knot $K \subset T \subset S^3$ parametrized by $\{(p\theta, q\theta) \mid \theta \in [0, 2\pi)\}$ in the $S^1 \times S^1$ coordinates of T. The figure below shows the case $(p, q) = (4, 3)$. Calculate $\pi_1(S^3 - K)$. *Hint: $S^3 - K$ decomposes into two solid tori with K removed from their boundaries.*



Algebraic Topology

(Unless indicated otherwise all homology and cohomology groups have integer coefficients.)

4. M is a closed, connected, orientable 5-manifold with $H_1(M) = \mathbb{Z}_7$ and $H_2(M) = \mathbb{Z}$. Compute $H_p(M)$ and $H^p(M)$ for all $p \geq 0$.

5. Let T^2 be the torus $S^1 \times S^1$. Let $T^1 = S^1 \times \{1\}$. (S^1 is regarded as the unit circle in the complex plane.) Compute $H_2(T^2, T^1)$ and $H_1(T^2, T^1)$. You may assume that T^1 and T^2 have the homology groups that they have with their standard generators.

THERE IS ONE MORE PROBLEM ON THE NEXT PAGE

6. Let X be the 2-complex obtained by identifying the boundaries of the two triangles D and E as indicated below. Give a careful, detailed computation of all the homology groups of X using cellular homology.