Geometric and Algebraic Topology
Comprehensive exam August 2011

Partial credit will be given for your work. Credit equivalent to five complete problems will guarantee a pass.

Geometric topology

1. List the two-sheeted covering spaces (up to covering space isomorphism) of the 3-torus $S^1 \times S^1 \times S^1$. Justify your answer.

2. Let F be a closed, orientable 2-manifold which is not the 2-sphere.

   (a) If A and B are non-separating, simple closed curves on F, prove there is a homeomorphism $h : F \longrightarrow F$ such that $h(A) = B$.

   (b) Find a counterexample to the theorem above if the hypothesis of non-separation is omitted.

3. Let F be a closed, orientable 2-manifold of genus at least 1, and let X be a simply connected space. Show that any map $f : X \longrightarrow F$ is homotopic to a constant map.

Algebraic topology

1. Let K be a compact 3-manifold with boundary, and let $T = D^2 \times S^1$ where D^2 is the closed unit disk, and S^1 is the circle. Suppose $K \cup T = S^3$ (the three-sphere), and $K \cap T = S^1 \times S^1$. Calculate the homology groups of K.

2. Let $R(P^2)$ denote the real projective plane. Compute the integral and mod 2 homology groups of $R(P^2) \times R(P^2)$.

3. Let $A \subset M$, and suppose the inclusion induced map $i* : H_n(A) \longrightarrow H_n(M)$ is one to one for each n. Prove that $H_k(M, A)$ is isomorphic with the quotient group $H_k(M)/H_k(A)$ for each k.