

Math 5313 Geometric Topology

1. Compact, connected 2-manifolds, n -manifolds, trees, graphs and Euler characteristic.
2. The fundamental group, retractions and deformation retractions, the fundamental group of a product space, homotopy equivalence and simple connectivity.
3. Basic combinatorial group theory: free groups, free products and presentations.
4. Seifert and Van Kampen theorem, computations of fundamental groups of compact, connected surfaces and CW-complexes.
5. Covering spaces, lifting theorems, regular covering spaces and quotient spaces of properly discontinuous group actions,
classification of covering spaces.
6. Applications of homotopy theory: the Brouwer fixed point theorem and the Borsuk-Ulam theorem in dimension two,
applications to knot theory.

REFERENCE: William S. Massey, A Basic Course in Algebraic Topology, GTM 127

Math 6323 Algebraic Topology I

1. Definition of singular homology groups, the exact sequence of a pair, homotopy invariance, excision property,
Mayer-Vietoris sequence, Eilenberg-Steenrod axioms.
2. Computations of homology groups of finite graphs and manifolds, the Jordan-Brouwer separation theorem, relation between
the fundamental group and the first homology group.

3. Homology of a CW-complex, simplicial homology.
4. Homology with arbitrary coefficients, the universal coefficient theorem.
5. Homology of product spaces, the Künneth theorem, the Eilenberg-Zilber theorem.
6. Definition of cohomology groups, the universal coefficient theorem, excision property, Eilenberg-Steenrod axioms,
the Mayer-Vietoris sequence.
7. Cup and cap products, computation of cup products in projective spaces.
8. Orientations for manifolds, Poincaré duality theorem, Alexander duality theorem, Lefschetz duality theorem,
applications of duality theorems.

REFERENCES: William S. Massey, Algebraic Topology: An Introduction, GTM 56; Marvin J. Greenberg and John R. Harper,

Algebraic Topology, a First Course; Edwin H. Spanier, Algebraic Topology, Springer-Verlag; J. Munkres, Elements

of Algebraic Topology; J. Vick, Homology Theory, GTM 145.