Potential Theory and Applications II

MATH 6010

Time and Place: TR 9:00-10:15 a.m. in MSCS 509
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Potential theory grew out of electrostatics and gravity theory. Distribution of charge on a conductor provides important insight and illustration to many results in potential theory. We shall give definitive answers to many classical questions of real and complex analysis such as solvability of the Dirichlet problem, characterization of removable sets for harmonic functions, integral representations of harmonic and analytic functions, exact bounds, etc. Further applications are found in approximation and interpolation of functions, number theory, numerical analysis, orthogonal polynomials, random matrices and polynomials, and far beyond. In the secomd part of this course, we shall consider more advanced topics and many applications.

Brief contents

1. Harmonic functions: Analytic completion, series expansion, uniqueness principle, mean-value property, maximum-minimum principles, Poisson integral formula, boundary values, reflection principle, Harnack's inequality and theorem.
2. Subharmonic functions: Majorization by harmonic functions, mean-value inequality property, sequences of subharmonic functions,

maximum principle, integrability.
3. Potentials: Maximum principle, continuity, weak* convergence of measures, energy problem, equilibrium measure, conductor potential, Frostman's theorem.
4. Capacity: Properties of capacity, transfinite diameter, Chebyshev constant, Evans' potential, generalized maximum principle, removable singularities of harmonic functions.
5. Dirichlet problem: Upper and lower functions, Perron's solution, regular boundary points, barrier, Green function, connection with conductor potential and conformal mapping, criteria of regularity, three-circle theorem, harmonic measure, two-constant theorem.
6. Applications: Inequalities for polynomials and analytic functions, zero distribution, etc.

Prerequisites: Complex and Real Analysis courses.

Selected references:
1. D. H. Armitage and S. J. Gardiner, Classical Potential Theory, Springer, New York, 2001.
2. N. S. Landkof, Foundations of Modern Potential Theory, Springer-Verlag, New York - Heidelberg, 1972.
3. T. Ransford, Potential Theory in the Complex Plane, Cambridge University Press, 1995.
4. E. B. Saff and V. Totik, Logarithmic Potentials with External Fields, Springer, New York, 1997.