Ordinary Differential Equations (Ph.D.)

Preparatory Courses: Math 4233, 5023

1. First-order ordinary differential equations (ODEs) : ODEs with separated variables, linear first-order ODEs, exact ODEs, logistic ODEs, Bernoulli's equation, Riccati's equation, and implicit ODEs.
2. Banach space, fixed point theorem for contractive mappings, the existence and uniqueness theorem for first-order ODEs with the Lipschitz condition, the Peano existence theorem for first-order ODEs, the existence and uniqueness theorem for first-order complex ODEs.
3. Systems of ODEs and high-order ODEs: fundamental matrix, Wronskian, the existence and uniqueness theorems.
4. Sturm-Liouville boundary-value problems, fundamental solutions, Green's functions.
5. Eigenvalue problems: Sturm-Liouville eigenvalue problem, eigenvalue theory for compact self-adjoint operators in a Hilbert space, applications to initial- and boundary-value problems for partial differential equations.
6. Stability theory: stability and instability theorems for linear and almost linear systems, periodic solutions, limit cycles, invariant sets, attractors, Lyapunov function, Lyapunov's second method, Poincare-Bendixson theorem.

REFERENCES:
1. Boyce, W. E. and DiPrima, R. C. *Elementary differential equations and boundary value problems, 8th ed.* New York: Wiley, 2005.
2. Coddington, E. A. *An introduction to ordinary differential equations.* New York: Dover, 1989.
3. Coddington, E. A. and Carlson, R. *Linear ordinary differential equations.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1997.
4. Guckenheimer, J. and Holmes, P. *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields.* Applied Mathematical Sciences, 42. Springer-Verlag, New York, 1983.
5. Walter, W. *Ordinary differential equations.* Graduate Texts in Mathematics, 182. Springer, 1998.