Math 6790: Equidistribution Theorems in Number Theory

Professor: Paul Fili
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Lecture: TuTh 3:30-4:45 in MSCS 509
Office: 532 Mathematical Sciences Building
Office Hours: TBA
Website: We will primarily use the Online Classroom (D2L) at https://online.okstate.edu/. Some information may also be posted at my homepage https://www.math.okstate.edu/~fili/.
Prerequisites: Familiarity with ring and field at graduate level, or permission of instructor.
Textbooks: None. Various useful papers will be posted on

About this course: Equidistribution has become a powerful tool for solving problems in Diophantine geometry and more recently in arithmetic dynamics. We will explore what it means for numbers to equidistribute and study important examples of equidistribution in various contexts. Equidistribution laws usually characterize "special points" in various systems, and knowing their distribution tells us something about the structure of the system. The celebrated proof of the Bogomolov conjecture for abelian varieties was proven by establishing an equidistribution result. This result inspired similar theorems in the arithmetic of dynamical systems which are now being used to prove dynamical analogues of these results, including applications to the problems of "unlikely intersections" between different dynamical systems.

We'll begin by defining what points are "special" by means of the notion of height on tori, elliptic curves, and dynamical systems, and then we'll discuss and prove in varying levels of detail the associated equidistribution results for points of low height (the theorems of Bilu, Szpiro-Ullmo-Zhang, Baker-Rumely and Favre-Rivera-Letelier) and some of the major applications of these results. Several open conjectures and questions will also be discussed.

Expected background: We will assume some basic algebraic number theory and real analysis.

Homework: Homework problems will be assigned throughout the semester and any questions may be turned in at any point during the semester. See the *Grading* section below for the relevant scoring information. You are allowed (and encouraged) to collaborate on homework so long as you do not copy each other's work (i.e., you can discuss the ideas and work together to find a solution, but you must write your own solutions without looking at other students' write-ups). You should feel free to discuss homework questions with me. Direct copying of other students' work is an academic integrity violation may result in an an F! grade for the course, so be sure to write up your own solutions.

Typesetting homework: Students are encouraged to download and use LaTeX (commonly pronounced "LAY-teck") to typeset their homework. LaTeX is used almost universally for writing in the fields of mathematics, physics, and computer science and is worth learning. Templates will be provided to ease learning of LaTeX, and you can easily find answers to many basic LaTeX questions on the web. For more information on typesetting software, visit the LaTeX project at http://www.latex-project.org. You will need to download and install first a "distribution" and then download a typesetting front-end which will use that distribution. Common choices for a front-end include Texmaker (cross-platform, what I use on my Macbook), Kile (available for Linux – my personal favorite), and LyX (available for Mac and Linux, as well as Windows under cygwin). LyX in particular is a WYSIWYG editor so it might be easier for first time users.

I am happy to help students set up LaTeX on their computers. Feel free to e-mail me for advice, or to bring your laptop to office hours for help setting up LaTeX.

Grading: 20% of your grade will be based on lecture attendance and 80% will be based on the homework. Homework problems distributed during the semester will have various weights; easy questions will often be worth 5 points, while more challenging or involved questions will be worth 10 or 15 points. You will be awarded partial credit for homework solutions. Your homework score will be totaled at the end of the semester, each 1 point of homework counting for 1% on your final score, up to a maximum of 80 points for the 80% of your grade which consists of homework.

Attendance: Attendance is required for this course. *R*epeated absences will result in losing credit for attendance (20% of your grade).

Students with disabilities: If you require any accommodations to assist your learning, you must first register with the Office of Student Disability Services, who will then inform me as to what accommodations to provide.

Syllabus Attachment: Please read the OSU syllabus attachment which follows this document.