Math 4950, Problem Solving Seminar

Course Information Fall 2017

Course Objectives: The objectives of the Problem Solving Seminar are to provide students who enjoy mathematics with experiences in problem solving that help you add some new mathematical skills to your existing repertoire, help develop your creativity, and enhance your ability to read and write mathematical arguments, all while having some fun and hopefully creating a killer Putnam team in the meanwhile. Some sample problems are on the last page of this handout.

Instructors: This course is team-taught with three instructors:

- ▷ Dr. Lisa Mantini, 410 MSCS, lisa.mantini@okstate.edu, 405–744–5777;
  - – Office Hours: M 3:30–4:30 PM, W 1:00-2:00 PM, R 2:30-3:30 PM, and by appointment.
- ▷ Dr. Ed Richmond, 427 MSCS, edward.richmond@okstate.edu, 405–744–5791;
  - – Office Hours: M 3:00–4:30 PM, R 9:00–10:30 AM, and by appointment.
- ▷ Dr. Detelin Dosev, 528 MSCS, dosev@okstate.edu, 405–744–5787.
  - – Office Hours: to be announced.

Course Times: Wednesdays from 3:30 until 4:45 PM in MSCS 405 and 428.

Prerequisites: Official catalog prerequisites for this course are Math 2233, Differential Equations, and Math 3013, Linear Algebra, but in practice the prerequisites probably are Calculus II (Math 2153), enjoyment of challenging problems, and consent of one of the instructors. Some experience with mathematical arguments as gained in Math 3613, Introduction to Abstract Algebra, is probably more useful than any specific knowledge of differential equations. Then again, Differential Equations is the course in which students really learn calculus, is it not? ⌣̈

Textbook: There is no required text. We will provide handouts if needed. Some nice resources on problem solving would include

- ▷ *Problem-Solving Through Problems*, by Loren C. Larson, Springer-Verlag, 1983, ISBN 0-387-96171-2.
- ▷ *The Art and Craft of Problem Solving*, 2nd edition, by Paul Zeitz.
- ▷ The archive of Putnam Exams and solutions at kskedlaya.org/putnam-archive.
- ▷ The Problems column in any of several mathematical journals, notably the *American Mathematical Monthy*, *College Math Journal*, and *Mathematics Magazine*, the three journals published by the Mathematical Association of America. All OSU students are entitled to membership in the MAA without additional charge as part of our institutional membership, and all members receive electronic copies of the journals included with membership. Let me know if you are interested.

Course format: This is a seminar course divided into three four-week segments, one taught by each instructor. Each segment will begin with weeks in which new problem-solving techniques are introduced. One to two problems will be assigned each week. Students should pick one of the assigned problems to try to solve each week. The class will discuss attempts at solutions during subsequent weeks and hints may be requested. Full or partial solutions may also be presented to the class, and drafts of solutions may be submitted to the instructors for feedback. Each segment will end with a Mini-Putnam Exam, which is an in-class problem solving session designed to simulate the experience of the Putnam Exam. Finally, students will select two problems from each segment, six overall, whose solutions will be submitted as one of the six required Reports in the course. These reports should contain a complete and correct solution to the problem, fully justified and written clearly enough that it could be submitted for publication.

Course Requirements: Your grade in the course will be based on attendance and your written reports. Students should plan to attend at least ten class sessions, missing no more than one session taught by any of the instructors. Please notify your instructor in advance of any absence. Attendance is worth 50 points, 5 points per session. You will also submit six written reports of fully solved problem solutions, worth 150 points, 25 points each. A total of 200 points are available.

Item	**Points Available**	**Total**
Attendance, 10 sessions	5 points each	50 points
Reports, 6 required	25 points each	150 points
Total		200 points

We expect that a grade of 85% or more will earn an A. Since each report is based on one problem only, and we are willing to provide feedback on earlier drafts before final report submission, we expect that each student has the potential to earn a grade of A. Please note that the Mini-Putnam Exams are not graded as typical exams. They are in-class problem solving experiences. Successfully solving any one problem during the in-class session is not required!

Putnam Exam: All students in the course, and any of your friends, are encouraged to participate in the Putnam Exam on Saturday, December 2, 2017.

> **We will allow participation in the Putnam Exam to earn 25 points towards one of your problem reports.**

The William Lowell Putnam Mathematical Competition is the preeminent mathematics competition for college students in the US and Canada (probably in the world!). It is given annually on the first Saturday in December in two sessions, from 9:00–12:00 noon and from 2:00–5:00 PM (central time zone). The competition dates back to 1938. It is an individual competition, and each student is eligible to participate at most four times. Each college with at least three participants may also designate three students to serve as their college team, with typically the ranks of the team members (not the numerical scores) serving to determine the team score. Typically more than

2000 students participate from hundreds of colleges and universities, and typically the median score on the exam is 0. Additional information is available at the web site at Santa Clara University, math.scu.edu/putnam, and a history of winning individuals and teams is available at the web site of the Mathematical Association of America at www.maa.org/programs/maa-awards/. The Art of Problem Solving organization also has additional information on its web site at artofproblemsolving.com.

Putnam Exam Recognitions: The highest ranking contestant from Oklahoma State University will have their name engraved on our plaque, mounted on the fourth floor of MSCS. In addition, the highest ranking contestant from the Oklahoma-Arkansas region will have their name listed on the web site of the MAA Oklahoma-Arkansas section and engraved on a plaque. Nationally, the five top contestants will be designated at Putnam Fellows, and the winner will win a one-year scholarship to Harvard University! Anyone who earns a Putnam Fellow designation earns an A in this course. Finally, there is an unofficial Big 12 competition among Putnam teams. OSU has not won this competition since the 1990's, I believe, but we'd be happy to reverse that trend.

Putnam Exam Schedule Notes: Interested OSU students might want to note that the first weekend in December is the likely weekend of the Big 12 Football Championship Game. There is some chance (or, more accurately, hope) that OSU will be playing in that game, which is scheduled to take place in Arlington, TX, likely on Saturday December 2, with the game time not yet being known. Students involved with the game will have to keep this in mind. The week of December 4–8 is also Pre-Finals Week at OSU. Final exams start on December 11, 2017.

Course Calendar: Here is an approximate course schedule.

Date	**Topic/Event**	**Due**	**Instructor**
August 23	Course introduction		All
August 30	Topic M1		Mantini
September 6	Topic M2		Mantini
September 13	Topic M3	Report 1	Mantini
September 20	Mini-Putnam Exam 1		
September 27	Topic R4	Report 2	Richmond
October 4	Topic R5		Richmond
October 11 and/or 18	Topic R6	Report 3	Richmond
October 25	Mini-Putnam Exam 2		
November 1	Topic D7	Report 4	Dosev
November 8	Topic D8		Dosev
November 15	Topic D9	Report 5	Dosev
November 22	Thanksgiving holiday		
November 29	Mini-Putnam Exam 3		
Saturday, December 2	**Putnam Exam**	514 MSCS, 9:00 AM–5:00 PM	
December 6	Wrap-up		All
December 13	no meeting	Report 6	

Academic Integrity: Oklahoma State University is committed to the maintenance of the highest standards of integrity and ethical conduct of its members. This level of ethical behavior and integrity will be maintained in this course. Participating in a behavior that violates academic integrity will result in your being sanctioned. These behaviors include, but are not limited to, unauthorized collaborations and plagiarism. Violations may subject you to disciplinary action including the following: receiving a failing grade on an assignment, examination or course, receiving a notation of a violation of academic integrity on your transcript (F!), or being suspended from the University. Sanctions are much more severe for graduate students — see academicintegrity.okstate.edu.

- ▷ With regard to this course, we encourage the discussion of problems and their solutions. However, you must write up your Reports and all assignments that you submit for this course *yourself* unless an assignment is specifically listed as a group assignment. You must never claim ideas that are not your own as your own. If you obtain significant help from an individual other than one of the instructors, that person should be cited in your Report's bibliography as Last Name, First Name, *Personal Communication*, with a brief description of the help received.
- ▷ All written sources that are influential in your final report, either from the internet or from the library, must be cited in your bibliography as well. We encourage you to try to solve the problems in this course yourself and not search for solutions on the internet. If you can find the solution on the internet, most likely so can we.
- ▷ When consulting a written source, you must make sure that you have come to understand whatever you read in your own way. Reports and problem solutions must never be copied verbatim but must be written in your own words. This means that you should close your book or browser and process the material on your own before writing it up on your own. If you don't understand an idea or could not explain it verbally to us, then it should not be included on anything you submit to us.

Sample Problem 1: Basketball star Shanille O'Keal's team statistician keeps track of the number $S(N)$ of successful free throws she has made in her first N attempts of the season. Early in the season, $S(N)$ was less than 80% of N, but by the end of the season, $S(N)$ was more than 80% of N. Was there necessarily a moment in between when $S(N)$ was exactly 80% of N?

Sample Problem 2: Let n be a fixed positive integer. How many ways are there to write n as a sum of positive integers, $n = a_1 + a_2 + \cdots + a_k$, with k an arbitrary positive integer and $a_1 \leq a_2 \leq \cdots \leq a_k \leq a_1 + 1$? For example, with $n = 4$, there are four ways: 4, $2 + 2$, $1 + 1 + 2$, and $1 + 1 + 1 + 1$.