Complex Analysis (Ph.D.)

Preparatory Courses: Math 5283, 5293

1. Complex number field $\mathbb{C}$, polar representation and roots of unity

2. Metric spaces

3. Topology of $\mathbb{C}$: Simple connectedness, connectedness, compactness, stereographic projection, and the spherical (chordal) metric

4. Analyticity and the Cauchy-Riemann equations

5. Elementary functions and their mapping properties: power, exponential, log, and trig functions

6. Elementary Riemann surfaces

7. Linear fractional (bilinear, Mobius) transformations, cross ratio

8. Complex integration: Line integrals, winding numbers, Cauchy (Cauchy-Goursat) integral theorem, Cauchy integral formula

9. Applications of the Cauchy theorems:
   - Cauchy estimates and Liouville's theorem
   - Maximum modulus principle and the Schwarz lemma
   - Morera's theorem
   - Taylor's series and the Identity theorem
   - Laurent's series
   - Argument principle, Rouche's theorem, and the Open Mapping theorem

10. Classification of isolated singularities

11. Behavior of a function near an isolated singularity

12. Residue theory and its use in evaluating assorted improper real integrals

13. Normal families, compactness in the metric space $\mathbb{H}(D)$, Montel's theorem

14. Riemann Mapping Theorem

15. Entire functions: Infinite products and the Weierstrass Factorization Theorem

16. Meromorphic functions and the Mittag-Leffler theorem
17. Analytic continuation, the Monodromy theorem, and complete analytic functions

18. Harmonic functions, Poisson integral, Harnack’s principle