Complex Analysis (Ph.D.)

Preparatory Courses: Math 5283, 5293

1. Complex number field C, polar representation and roots of unity

2. Metric spaces

3. Topology of C: Simple connectedness, connectedness, compactness, stereographic projection, and the spherical (chordal) metric

4. Analyticity and the Cauchy-Riemann equations

5. Elementary functions and their mapping properties: power, exponential, log, and trig functions

6. Elementary Riemann surfaces

7. Linear fractional (bilinear, Mobius) transformations, cross ratio

8. Complex integration: Line integrals, winding numbers, Cauchy (Cauchy-Goursat) integral theorem, Cauchy integral formula

9. Applications of the Cauchy theorems:

- Cauchy estimates and Liouville's theorem
- Maximum modulus principle and the Schwarz lemma
- Morera's theorem
- Taylor's series and the Identity theorem
- Laurent's series
- Argument principle, Rouche's theorem, and the Open Mapping theorem

10. Classification of isolated singularities

11. Behavior of a function near an isolated singularity

12. Residue theory and its use in evaluating assorted improper real integrals

13. Normal families, compactness in the metric space H(D), Montel's theorem

14. Riemann Mapping Theorem

15. Entire functions: Infinite products and the Weierstrass Factorization Theorem

16. Meromorphic functions and the Mittag-Leffler theorem

17. Analytic continuation, the Monodromy theorem, and complete analytic functions

18. Harmonic functions, Poisson integral, Harnack's principle

REFERENCES: L. V. Ahlfors, *Complex Analysis*; R. B. Ash, *Complex Variables*; R.V. Churchill and J.W. Brown, *Complex Variables and Applications*; J. B. Conway, *Functions of One Complex Variable*; E. Hille, *Analytic Function Theory,* Vols. I & II; K. Knopp, *Theory of Functions,* Parts I & II; L. L. Pennisi, *Elements of Complex Variables*; W. Rudin, *Real and Complex Analysis,* (chapters 10-16); D. Ullrich, *Complex Made Simple*.