An Inquiry-based Approach to Geometry:
Study Teams

Overview

In other places in this instructor's guide, mention is made of using a *group approach* for the delivery of instruction. In Part I, we discuss in depth using a *study teams approach* for the delivery of instruction. Because the term "group" is widely used – and actually not used with much degree of consistency – the place to begin is to investigate the meanings of these terms. The following are discussed: *What is a Learning Group; Some Logistical Issues Concerning Study Teams; Characteristics of Effective Study Teams.*

Part II, beginning on p. 6, is the discussion of the ***GeoSET Study Team Model***. The actual templates for classroom use are discussed and provided for immediate copying. Midterm and final assessment instruments concerning this model are also provided. If you wish to employ a study teams approach, please use these materials. Feel free to change them – and make them better!

Information from the research literature on the history and use of learning groups, and the related role of mathematics in schools are discussed in the appendices for those interested in knowing "how we got where we are today."

Part I

What is a Learning Group? We begin the journey of trying to characterize a learning group by looking back at the historical developments associated with it. Beginning in the 1980's, educators began to define more comprehensive terms related to learning groups. We find that a distinction was made between cooperative learning and collaborative learning. According to Ted Panitz, cooperative learning is "a structure of interaction designed to facilitate the accomplishment of a specific end product or goal through people working together in groups" while collaborative learning is defined to be "a philosophy of interaction and personal lifestyle where individuals are responsible for their actions, including learning, and respect the abilities and contributions of their peers" (Panitz, 1997).

Groups involved in collaborative learning are totally responsible for solving the problem at hand. They must decide whether they have enough information or be able to find the information from other sources. The process of finding the information is a group effort, and the group members assign responsibilities to each other. The teacher is in the background; only there to ask the group for occasional reports on progress, help with

group conflicts, etc. The group itself determines the final solution on their own. Through this process of collaboration, students develop a strong ownership of the solution (Panitz, 1997). In contrast, groups involved in cooperative learning still work in groups to complete the problem at hand, but they are provided with all of the information that is required. The students do the work essential to solve the problem, but the teacher controls the process at each stage; the teacher still has complete control of the class and may require the solution be presented in a specific manner. The main difference between the collaborative learning and cooperative learning models is that the former is more student-centered while the latter is more closely controlled by the teacher (Panitz, 1997).

As we began to become involved in considering using some methodology that empowered students to be more responsible for their learning, we encountered issues like these; what do these commonly used terms mean and connote in the learning process? For our purposes, we wanted to develop learning communities that nurture student interaction and dependence; we wanted them to be able to trust each other's thinking and not to rely on the teacher for their knowledge.

We were also concerned with the overworked, and seemingly misunderstood, term "group" to describe our evolving delivery methodology. The term "Groups" is used in many different ways to convey many different responsibilities to students. Phrases like: Get into groups during class and … ; work with your neighbor in class and … ; compare your work with the members of your group after class. Surely, you can add some others. We went to the literature for direction. Barbara Davis has categorized groups as informal learning groups, formal learning groups, and study teams (1993).

Briefly,

- ***Informal learning groups*** are "temporary clusterings of students within a single class session" (Davis, 1993). This categorization involves a student turning to his/her neighbors and discussing a specific topic or problem; little connection between students is made largely because there is rarely enough time to do so. Seldom do learning relationships evolve from this type of grouping; the lifetime of this group is simply until the end of the class period for a specific day.
- ***Formal learning groups*** are "established to complete a specific task" (Davis, 1993). This categorization involves students being assigned to a group for the purpose of completing a prescribed task. Because the group spends more than just the class period together, there is more opportunity for them to learn from each other and begin to understand each others thinking. The lifetime of this group is determined by the time required to complete the task; once the task has been completed, the group dissolves.

- ***Study teams*** are semester-long assigned clusterings of students within a single class setting. There is "stable membership" and members "provide other members with support, encouragement, and assistance in completing course requirements and assignments" (Davis, 1993).

From Davis' categorization, it seems clear that pedagogical model employing study teams has the potential to contribute more to students' learning than either a model employing informal or formal groups. The longevity of study teams allows for cohesiveness and a deeper level of bonding between group members. When this occurs, there is a desire to help one another succeed in the course. Group members become more adept at communicating with each other mathematically.

Our goal in this geometry course for prospective elementary and middle school teachers was to create an inquiry-based curriculum and learning environment that supported each student's growth toward being a confident, independent learner empowered with the help of peers to be able to make sense of the geometric world. Inherent in an inquiry-based approach involving peer interactions is nurturing collaborative learning; our approach involved study teams. The development of a collaborative learning model involving study teams was no small task and, in fact, when realized violated most students' beliefs about how mathematics is supposed to be taught and learned. Learning from your peers in settings within (and outside of) the classroom without your teacher ultimately providing the "stamp of approval" or telling you the "things you need to know" to move the investigation along is a very uncomfortable feeling.

Finally, because most students have not been involved previously in their classes with using "study teams" as such, they do not have predisposed ideas about what their study team is supposed to do. The word "group" is specifically avoided throughout because of the plethora of possible meanings it connotes; group has been replaced by "study team".

Some Logistical Issues Concerning Study Teams. We turned to the research literature to guide us with attending to such logistical issues as: What is the size of a study team? How should study teams be formed?

Relative to the size of a study team, we found that the number varied from only two students to up to six students with the general consensus being four members. Dubinsky, Hagelgans, Reynolds, Schwingendorf, Shahin, Vidakovic, and Wimbish recommend that three or four students (1995). The Institute for Interactive Media and Learning (IML, 2006) agrees that teams of around four members tend to work well. There seem to be several reasons why teams of four are recommended throughout literature. The number of members is small enough that students have less conflicts when organizing meetings. Also, with only four members, it is more difficult for a student to be left out of the necessary work – students do not fall through the cracks as often. Also, with a smaller number of opinions, consensus can be reached easier (IML, 2006). Teams of three were recommended as well, but with the drawback that they are more affected by the absence of any one of the members. Whereas a team of four can continue to function as a team if one student is absent or drops out (Dubinsky, Hagelgans, Reynolds, Schwingendorf, Shahin, Vidakovic, Wimbish, 1995).

Relative to forming a study team, much of the literature identified four main selection methods: random assignment, self-selection, selective assignment, and task

assignment. Note. Because the literature did not make a distinction between a group and a study team, the more general term is used below.

- *Random Assignment.* Typically, the teacher randomly places each student with other students according to some scheme, e.g., counting off students by numbers. Random assignments are fairly quick and simple for the teacher, break up friendship groups, and sometimes allow people to work together who normally would not (IML, 2006).

- *Self-selection.* The teacher simply asks students to form groups themselves. Students usually select others who they know or have worked with before in other classes. If a student does not know anyone else in the class, then he/she usually forms a group with those sitting in close proximity.

- *Selective Assignment*. This approach takes self-selection a step further. It allows students to choose their own groups, but attempts to get the students to select members based upon specific criteria. Some examples of criteria could be similar goals, similar schedules, or learning styles. These criteria could be used to form homogenous groups with everyone sharing the criteria, or heterogeneous where the group is more balanced with different aspects of the criteria.

- *Task Assignment.* With this method of selection, the students have the opportunity to select a topic from a list created by the teacher. Groups are then formed based upon the chosen topics; students who have expressed a special interest in a particular topic are grouped together to work on the selected task. This method might work best for a short-term project rather than a long-term one.

In our situation, we chose the method of selective assessment. Prior to students selecting their study teams, the expectations and requirements of the course were described and discussed. Then the notion of a study team is introduced as the vehicle by which success in the course will be facilitated. In selecting their study teams, students are encouraged to consider such issues as: the role of the study team in helping me succeed in the course; my strengths and weaknesses as a team member; my expectations of the other team members; the roles and contributions that team members will play in the functioning of the team. Once selected, study teams are encouraged to develop techniques for managing their day-to-day functioning, e.g., meetings outside of class, Study Team Journal entries. Early evidence of positive team morale and bonding can been found in team names, e.g., Super Squares, Origami Mamas, and Pink Ladies.

It is very difficult for students to make the transition from the traditional learning environment where the teacher is the source of information to one in which their growth toward being confident, independent learners empowered with the help of peers. It is essential for teachers to be patient and nurturing throughout the semester. The rewards will come when the students come to value learning from their peers. There is research evidence that learning from peers does contribute to individual understanding. Sharon Hamilton (1999) believes that “working together results in greater understanding

than would have occurred had the students worked independently." Students each bring unique encounters and experiences with mathematics to the group setting. Within a group, students can pool this information to clarify ideas and learn from one another. Hamilton (1999) also believes that "spoken and/or written interactions contribute to this increased understanding." Because the student is not passively listening to the instructor explain a concept, he is required to be more active and involved in the learning process. In addition to actively listening to his group members, a student must speak mathematically to them and be able to communicate his meaning through writing. This group member is engaged in discussion and takes responsibility for his own learning. Good, Mulryan, and McCaslin found that "small-group instructional models can facilitate student achievement… as well as more favorable attitudes toward peers and subject matter" (1992). Not only are groups useful for curriculum understanding, but also groups can affect students' emotional and social views of their peers and the material.

Characteristics of Effective Study Teams. The guidelines for successful study team functioning that we developed were the result of careful scrutiny of the research literature on effective groups. Bakker, Fennimore, Fine, Pierce, and Tinzmann (1990) suggest that learning groups should set goals in order to be effective. This helps students to work together to accomplish their goals. They are also able to assess their work and check to see if they achieved their goals. Similar to setting goals, groups should define responsibilities to each of the members. These methods help to keep the group members accountable to one another.

Sarah-Marie Belcastro (2004) mentions several more attributes common to successful groups. These include all group members being present, members being prepared when they arrive, discussing problems until a consensus is reached, and the group encouraging multiple approaches. The attendance of group members includes class attendance and also attendance to the scheduled group meetings outside of class. Before group members show up to either type of meeting, they should do "some background work before they can collaborate effectively" (Beachy, 2001).

When a problem arises, students should have an "active exchange of ideas and …engage in discussion" (Southard, 1999). When students are actively exchanging ideas, they should be closely listening to other members of the group, comparing what has been shared to their own knowledge, and replying in context. This discussion should continue until a consensus is reached concerning the problem at hand. When a consensus is finally reached, different members may have reached the same point using different avenues, especially in mathematics.

Group members must realize that multiple approaches are possible. A successful group will even realize that it is important and helpful to learn multiple approaches when solving problems. Students in a group must learn to value one another's opinion before they will ever begin to listen to each other. Through this, individual learning results from the group process (Southard, 1999).

According to Slavin (1989), in order to have effective group learning, there must be "individual accountability." Students should hold each other accountable and occasionally the teacher must hold each student accountable. This accountability between group members brings about another characteristic common to successful groups, students must "perceive that they sink or swim together, that each member is responsible to and dependent on all the others, and that one cannot succeed unless all in the group succeed" (Davis, 1993).

Finally, Herzig and Kung (2003) proposed six variables used to measure how students view aspects related to mathematics. The first four of these were adapted from the Fennema-Sherman Mathematics Attitude Scales (Fennema and Sherman, 1986). The last two were created for a study conducted by Herzig and Kung. The six variables include:

- Confidence in Learning Mathematics
- Attitude Toward Success in Mathematics
- Mathematics Usefulness
- Effectance Motivation in Mathematics
- Beliefs About Mathematics
- Learning With Others

In a study conducted by Richardson (Spring 2006) as part of the GeoSET assessment activity, study team member comments were analyzed according to these six variables; evidence was sought that study teams positively affected these six variables. Results of this study are available upon request.

Part II

Background and Description of the GeoSET Study Team Model. Geometric Structures (MATH 3403) is a 3000-level mathematics required of all prospective elementary and middle school teachers at Oklahoma State University. Several sections of this course are taught each semester with classes meeting twice a week; each class meeting is 75 minutes. Based on years of experience with the traditional elementary geometry curriculum and delivery methods, we began an adventure that we hoped would change tradition. Our efforts culminated in a multi-year grant from the National Science Foundation to develop a creative, inquiry-based experience with geometry that would be appropriate for prospective elementary and middle school teachers. We would like to stimulate and draw out each student's curiosity. We would like to support each student's growth toward being a confident, independent learner empowered with the help of peers to make sense of the geometric world.

The inquiry-based activities that comprise the curriculum are designed to minimize the need for traditional instructor lecturing. Classroom discussions and group activities provide a rich and essential resource to motivate student's efforts to make sense of the course material. The instructor's ability to manage these discussions and activities without traditional lecturing is a critical factor in the success of this approach. With this approach, we have found that students arrive at a deeper, more robust, understanding and appreciation for the power, beauty and meaning of geometry.

A constant and varied stream of geometry problems is provided by these activities. Student sharing of their thinking and experiences with these problems provides many insights into how and why different methods work for solving problems. We have been gratified with the fact that, although there is little drill and practice homework presented in these activities, students seem to become as good at working standard problems as they do in more traditional courses. We do find that students in this curriculum are definitely more adventurous in attempting unorthodox problems as well as stronger at explaining why the methods they choose make sense.

The ability to express geometric ideas verbally, physically and in writing and drawing is a valuable communication skill for teachers. Many activities in this text are designed specifically to provide opportunities to exercise these skills.

Managing Aspects of the GeoSET Study Team Model.

A. Course Syllabus. Making student expectations known from the outset is necessary – students expect it! Since the notion of a Study Team is undoubtedly new to the students, you must describe in detail how their involvement in Study Teams will ultimately help them be successful and influence their grade. I offer several statements below for your consideration and inclusion directly in your syllabus.

It is also important to nurture student accountability throughout the semester – accountability to one's self and to the other members of the study team.

- *Syllabus Inclusion 1 –Daily Routine*
  This statement prepares students for how the class sessions will be conducted and the expectations for out of class work.

  Daily Routine. An assignment (about 3-5 Activity Pages) will be given at the end of nearly every class meeting. This completed assignment (homework) is due *at the start of* the next class meeting. Selected Activity Pages from the homework will be collected at the beginning of the class meeting; be sure each assignment has been completed *before* you come to class. Work that is completed during the class meeting (classwork) will be collected regularly. To be prepared for each class meeting, you should bring your homework and class activities as well as the manipulative currently being used (you will be told what it is).

- *Syllabus Inclusion 2 –Daily Grade*
  This statement sets the standards quizzes, homework, and classwork.

  Daily Grade. The daily grade includes quizzes, homework assignments, and classwork. Assignments will not be accepted late *for any reason whatsoever*. You must be present for the entire class meeting to submit any work that is due that day (homework, classwork, projects, etc.) or to present a quiz; *there are no quiz make-ups for any reason whatsoever*. For very special situations that are *approved in advance*, the instructor reserves the right to allow a quiz to be presented early.

- *Syllabus Inclusion 3 –Project Grade*
  This statement addresses out of class projects.

  Project Grade. During the semester, several out of class projects will be assigned. These are fun! Take them seriously and *Be Creative*! You must be present for the entire class session to submit a project when it is due. Projects will not be accepted late *for any reason whatsoever*. For very special situations that are *approved in advance*, the instructor reserves the right to allow a project to be submitted early.

- *Syllabus Inclusion 4 –Study Team Grade*

  This statement introduces the Study Team notion; very important. It spells out explicitly what needs to be done NOW. This statement should be read, and reread, in class several times to emphasize the importance of the Study Team in this learning environment.

  Study Team Grade. Learning from your classmates is a proven ingredient for success. We believe in this and have arranged the course delivery accordingly. Each of you will have an opportunity during the first week or so to identify others with whom you wish to work this semester; a ***study team*** will ideally be composed of four students. Your first assignment: Identify classmates for your study team who *you can help learn* and who *can help you learn*. Each study team member has a responsibility to the study team's success and the study team collectively has a responsibility to each member's success! Establishing and nurturing study team spirit is vital to your success.

  Once formed, the study team should consider such issues as: (1) its goals and objectives (what each of you want from the study team); (2) the strengths each member brings to the study team and how they will be utilized (what role members will play - identify who does what); (3) how the study team will function (when do you meet outside of class, what you will do at study team meetings, and how you will help each other). As you contemplate these issues, others will arise; they are supposed to!

  Each study team will maintain a **Study Team Journal** – or simply a **JOURNAL** - that will be reviewed by the instructor periodically; definitely at exam time. Previous students have found that a standard 3-ring binder works very well for this purpose. Each team member is expected to contribute to the JOURNAL and identify his/her contributions. Your study team's journal is an evolving document over the course of the semester. Bring your Study Team Journal to each class meeting.

  Each study team member will be assigned the same Study Team Grade at the end of the semester. This grade is based on the team's level of active participation in class meetings throughout the semester and the quality of the team's Journal.

B. Supplemental Handouts.

- *Supplemental Handout 1 – Setting the Stage for the Study Team Journal*
  During the first or second week of the semester, provide Study Teams with copies of this handout. Review it often.

  Study Team Journal. Building a classroom community where students are nurtured to rely more on themselves in determining whether something is mathematically correct is a primary goal of this course. Toward accomplishing this goal, we rely on *Study Teams* of classmates. Each of you has had an opportunity to identify others with whom you wish to work this semester; each Study Team is composed of approximately four students.

  The Study Teams should now consider such issues as: (1) its goals and objectives (what you want from the Study Team); (2) the strengths each member brings to the Study Team and how they will be utilized (what role members will play - identify who does what); (3) how the Study Team will function (when do you meet, what will you do, and how you will help each other). As you contemplate these issues, others will arise; they are supposed to!

  Each Study Team will maintain a Study Team Journal that will be reviewed by the instructor periodically; definitely at exam time. Each member is expected to contribute to the journal and identify his/her contributions. *Your Study Team journal is an evolving document over the course of the semester*.

  Developing the Study Team Journal. Because items will be added periodically to your journal, it has been suggested that a standard 3-ring binder be used. Besides, a creative cover can be designed showcasing your Study Team's spirit!

  The ***first page*** of the Study Team Journal should include the *Study Team name* and *each Study Team member's name and information related to how to contact the Study Team member* [Note. Each Study Team member should have a copy of this sheet to facilitate communication among the members of the Study Team.].

  Following the first page, each Study Team member should identify and insert his/her written responses to the two statements on page 5 of the syllabus. These are:

  a) Write a brief statement (a paragraph or two)

stating your goals; include the course grade your are aiming for.

b) List the day-to-day actions you plan to take to achieve your goals.

You should check this list periodically during the semester to see if you are on course.

The Study Team should discuss each member's responses to these statements and formulate a Study Team *plan of attack*; this is ***page two***. There should be Study Team summary statements addressing the following issues:

Study Team goals and objectives (what will the Study Team help members do?)

Identification of each member's responsibility to the Study Team (who will be doing what?)

Describe how the Study Team will function (when will the Study Team meet? What will the Study Team do when it meets?)

What are the necessary ingredients of a Journal Entry? Each time the Study Team functions (for example, the Study Team meets for a purpose), there should be an entry in the Study Team's journal. Each entry is on a separate page in the journal. The *required information* to accompany each entry is:

- Date/Time of Study Team Function
- Identification of Participants
- Identification of Scribe (person writing the entry)
- Statement of the Purpose of the Study Team Function (What is trying to be Achieved?)
- Description of What the Study Team Did to Achieve the Purpose

Note. Some Study Teams have found that after they discuss the key mathematical idea(s) in an activity, they write them down in their journals. So, one element of a journal entry is a *statement summarizing the mathematical ideas that were investigated*. It is important that journal entries discuss/summarize mathematical ideas; another way to help you develop your ability to communicate mathematical ideas. Besides, reviewing rich journal entries will be helpful in studying for the exams !

- *Supplemental Handout 2 – Midterm Individual Assessment*
  This midterm formative assessment will enable you to determine how well the Study Teams are functioning. Review the individual responses and visit with Study Teams as a whole as needed to motivate action.

MIdterm Individual Assessment

Name: **Group:**

Please respond to each of the questions below; your comments will be considered in formulating suggestions that will be made to the study teams.

1. **Rate** each of the following ***on a scale from 1 to 10*** (10 being "best"):
   a. The *importance of your study team* in contributing to your success in the course. _____

   b. *Your contributions* to the study team _____
   .
   Explain in the space below *specifically what YOU have done* to justify your rating.

   Identify other contributions YOU plan to make.

   c. *How well* your study team is functioning. _____
.
   Identify in the space below *specific strengths and weaknesses that need to be addressed*.

   Identify the *actions* in your study team that YOU plan to take to overcome these weaknesses

2. Comment on how you are progressing in *accomplishing your goals* for the semester in this course.

3. Describe ***in detail*** what *actions* you plan to take to accomplish your goals for the semester

- *Supplemental Handout 3 – Final Individual Assessment*
  This final formative assessment will enable you to determine how well the Study Teams functioned. They are an excellent resource for course modifications in the future.

Final Individual Assessment

Name: **Group:**

Please respond to each of the questions below; your comments will be considered in formulating suggestions that will be made to the groups.

1. **Rate** each of the following ***on a scale from 1 to 10*** (10 being "best"):
   a. The *importance of your study team* in contributing to your success in the course. _____

   b. *Your contributions* to the study team _____
   .
   Explain in the space below *specifically what YOU have done* to justify your rating.

   c. *How well* did your study team function? _____
   .
   Identify in the space below *specific strengths and weaknesses that were NOT addressed*.

2. Comment on how well you *accomplished your goals* for the semester in this course.

3. a. Describe *what did you liked MOST* about our approach to study teams in contributing to your success in the course.

   b. Describe *what did you liked LEAST* about our approach to study teams.

   c. What suggestions do you have making the study team approach a more effective element of contributing to your success in the course?

C. Some Final Comments. I want to make a comment about Study Team "spirit". Teams that have bought into this model generally exhibit behaviors that are fun loving and involved. You can clearly see this when you are mingling with them on a regular basis. Whatever you can do to encourage a positive, caring, and sharing attitude among teams will pay off in the end. One of the easiest ways to encourage this attitude is to have the

teams select original names. The “Origami Mamas” and “Southern Sweeties” stick out in my mind as teams that saw value in working together from the beginning.

You really can’t write enough supporting comments on Daily Assignments, Quizzes, and Notebooks. Instead of making evaluative comments, ask questions that cause reflection like “How did that make sense to you?” I assess Journals at least four times during the semester. I prepare a cardboard cover sheet for each Journal; it is placed as the top page inside the cover. I write my dated comments on this page for me and them to read; I look for progress in addressing the issues raised.

I ask Study Teams to meet at least twice each week outside of the regular class sessions. The minutes of their meetings are included in the Team Journal for me to evaluate. I have noticed that some groups choose to meet immediately before the next scheduled class meeting; I have found that teams who met like this were less successful. I encourage students to meet at some time not associated with the regularly scheduled class meetings.

Final Notes.
1. Special recognition is due J. Stone for contributions made to this paper as part of the completion of the Creative Component of the master’s degree in mathematics at Oklahoma State University.

2. Partial support for this work was provided by the National Science Foundation grants #0231161 and #9752288. Any opinions, findings and conclusions or recommendations expressed in this work are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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APPENDICES

Appendix A: Some History on Learning Groups

The idea of using learning groups as a method of instruction can be traced in the literature to as early as 1917 with Sterling Leonard who said, "Students should be knit into a social group organized for mutual help and be aided to move steadily forward in an arduous way of attaining effective expression," (Hamilton, 1999). John Dewey also had thoughts about students working together when he said, "Learning should be motivated by the moving spirit of the whole group as a class is held together in participation of common activities," (Hamilton, 1999).

Learning groups were affected by the philosophies of progressive education during the late nineteenth to mid-twentieth centuries. John Dewey, commonly referred to as the founder of the progressive movement, believed that students learned following a five-step process, which he named the method of intelligence:

Become aware of the problem.
Specify the problem.
Introduce a hypothesis that, if correct, would solve the problem.
Assess the possible consequences of the proposed hypothesis.
Test the hypothesis experimentally. (Garrison, 1999).

Proponents of this movement wanted schools to better prepare students for a democratic society. To them, democracy was, "active participation by all citizens in social, political, and economic decisions that will affect their lives" (Garrison, 1999). In order to educate students to this level, they believed that teachers should develop a respect for diversity and develop critical, socially engaged intelligence. The latter would help students to collaboratively work together in the efforts of reaching a common goal (John Dewey Project on Progressive Education, 2003).

One of the leading advocators for progressive education was William Heard Kilpatrick. He believed that "subjects should be taught to students based on their direct practical value" (Klein, 2003). By restricting education to practical skills, the amount of content that was taught was considerably limited. This helped to "justify the slow pace of student centered, discovery learning, the centerpiece of progressivism" (Klein, 2003). In 1920, Kilpatrick published a report, The Problem of Mathematics in Secondary Education, where he continued to claim that nothing in mathematics should be taught unless its probable value could be shown, and he also "recommended the traditional high school mathematics curriculum for only a select few." He believed that algebra and geometry were not being taught to too few students, but to too many students (Klein 2003).

Not all educators shared in this belief. Mathematicians of this time were outraged and the Mathematical Association of America (MAA) responded by forming the National Committee on Mathematical Requirements who wrote a report called The Reorganization of Mathematics for Secondary Education (1923), referred to as the 1923 Report. The report discussed many topics related to school mathematics including issues related to the psychology of learning mathematics; some may be shocked to learn that this community

was concerned with the intended audience for school mathematics. The report supported the study of mathematics in terms of its applications, and also defended learning mathematics for its intrinsic value as well. Opposite of Kilpatrick's report, the 1923 Report focused on the importance of teaching mathematics to "every education person" (Klein, 2003).

Over the next few decades, the chasm between school curriculum content and pedagogy grew through movements such as the Life Adjustment Movement in the 1940's; what should be taught and to whom was a subject of great debate. Following WW II, the "nation witnessed tremendous scientific and engineering advances" such as "radar, cryptography, navigation, atomic energy, and other technological wonderments which changed the economy" and brought back "recognition of the importance of mathematics education in the schools" (Klein, 2003). The launching of Sputnik in 1957 by the Soviet Union also had sobering effects on the mathematics and science education of United States high school and college graduates. By comparison, the United States was behind and it became clear that it was in the "national interest to change our approach to education, in particular the curriculum of mathematics and science" (Bybee, 1998). This rethinking of the education of young people motivated the fall of progressivism. The importance the student's conceptual understandings of mathematics, not rote memorization, became the cornerstone for developing school curriculum.

Appendix B: The Role of Mathematics in Schools

The period in mathematics education from 1955 to 1975 became known as the New Math Era. "New Math" was not a panacea nor was it without its critics. But, it was "clearly a move away from the anti-intellectualism of the previous half-century of progressivist doctrine" (Klein, 2003). Generally, activity focused a great deal during this period on the curriculum and content that should be taught in the schools rather than the pedagogy. School curricula developed under many federally funded grants were "primarily intended for high school students whose career path included engineering and other science-related occupations" (Jones, 1991). Because of their unfamiliarity with these merging mathematics curricula, the American public was critical of "new math". The backlash came in the form of the "back to the basics" movement in the 1970's.

In the early 1970's, high school graduates of the "new math" era were beginning to enter college. What happened in colleges as a result was effected by the baby boom generation and such societal factors as open admission policies. Colleges were faced with bigger enrollments than they had ever faced before. Larger class sizes and less interaction with professors was the obvious short term solution to this problem. According to Herzig and Kung (2003), in the 1980s, an estimated 50% of students who chose science, mathematics, and engineering majors when they entered college switched to non-science majors. A study was designed to explain why so many students left these majors; one of the study's major findings was that the switchers and non-switchers were not different types of students. In fact, they were actually very "similar in their abilities, attitudes, high school preparation, willingness to work hard, and even their grades" in courses

related to their major. The largest difference, however, was that non-switchers had found several strategies they employed to survive. One of the strategies they mentioned that helped them out was working in peer groups (Herzig, 2003). College faculty attributed the cause for the declining attitudes towards mathematics to "inadequate facilities, large classes, and poor pre-college preparation of students" (Herzig, 2003). They primarily began to look for ways to better prepare the students as an alternative to "traditional classroom learning and teaching" (Gala, 1999). This prompted instructors to experiment with different methods of teaching mathematics. In the late 1970s, "cognitive science - the study of the mind" became a focus of research. As a result, the view of mathematics changed from "numbers and computations" to "math as problem solving" (Bybee, 1991). This view of mathematics as problem solving has the inherent underpinning of encouraging students to work collaboratively.

Capitalizing on the "good and the bad" of the previous twenty-five years, the National Council of Teachers of Mathematics (NCTM) published the first of its benchmark reports An Agenda for Action in 1980 which "called for new directions in mathematics education" (Klein, 2003). The Agenda proposed that problem solving be implemented as a substantial part of school mathematics. NCTM also advised that "teachers should use diverse instructional strategies, materials and resources" (1980). One of the instructional strategies recommended was small-group work. Klein (2003) summarized the report saying, "Team efforts in problem solving should be common place in elementary school classrooms." The chasm between curricular content and pedagogy was decreasing.

Focusing more on pedagogical issues brought a "renewed interest in cooperative group work" (Herzig, 2003). Several colleges began trying peer tutoring. "Peer tutoring and similar modes such as peer evaluation and classroom group work could be classified under the term: collaborative learning" (Gala, 1999). Simply having students work together during class time in learning groups is not enough. It is important to realize that "learning tasks need to be structured to ensure interaction among students, and classroom norms need to be established that support that interaction" (Herzig, 2003). According to Good, Mulryan, and McCaslin, "small-group interaction can be used in inappropriate was well as appropriate ways" (1992).

Appendix C: More Recent Publications Affecting the Use of Learning Groups

Within the last twenty-five years, many reports and papers have been published by the professional community that focuses on the use of learning groups in the classroom. A few are listed below.

- National Research Council (NRC) published *Everybody Counts – A Report to the Nation on the Future of Mathematics Education (1989). Everybody Counts* stated that mathematics education must undergo certain transitions in order to meet the challenges of the day; "the teaching of mathematics is shifting from an authoritarian model based on 'transmission of knowledge' to a student-centered practice featuring 'stimulation of learning' " (NRC, 1989). It also recommended many changes concerning the learning

environments such as: “encourage students to explore, help students to verbalize their mathematical ideas, show students that many mathematical questions have more than one right answer, teach students through experience the importance of careful reasoning and disciplined understanding, and build confidence in all students that they can learn mathematics” (NRC, 1989).

- NCTM published the *Curriculum and Evaluation Standards for School Mathematics (1989).* The *Standards* is comprehensive and can be looked at as a formula for school mathematics. It calls for “five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically” (1989). Another significant recommendation made by this publication is that “instruction should vary and include opportunities for – group and individual assignments…and discussion between teacher and students and among students” (NCTM, 1989).

- NCTM took the next step by publishing the *Professional Standards for Teaching Mathematics (1991).* NCTM believed it was necessary to state that classrooms must change; they must “shift toward classrooms as mathematical communities and away from classrooms as simply a collection of individuals, shift toward logic and mathematical evidence as verification and away from the teacher as the sole authority for right answers, and shift toward conjecturing, inventing, and problem solving and away from merely emphasizing finding the correct answer” (1991).

- NCTM took the step that brought us into 21st century by publishing the *Principles and Standards for School Mathematics (2000).* The *Principles* is a revision of the earlier document Standards (1989). Principles describes five Process Standards that “highlight ways of acquiring and using content knowledge” (2000). These are Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.

Used appropriately, learning groups is a methodology that facilitates problem solving, reasoning and proof, communication, connections, and representation. When a teacher uses learning groups appropriately, students explore and communicate their individual ideas with one another. We believe group work can result in the students each having higher confidence in their own mathematical abilities. Learning groups allows for students to work collaboratively on problem solving. While working together, students can learn to communicate their ideas efficiently and can help one another discover if their methods are mathematically sound. Learning from one another occurs independent of the teacher; the teacher is a facilitator.