

Chapter Outlines: Core and Optional APs

The text *Geometric Structures* includes far more material than can be covered in a one semester course. We hope that this document will be an aide to teachers in planning which activity pages (APs) to cover and which to skip. A chart for each chapter follows this introduction. These charts identify *core* APs, *optional* APs, and *student information* APs for each chapter.

The *core* APs provide coherent coverage of the central concepts of each chapter. Students reading and doing this sequence of APs, one or two per class period followed by a class discussion of issues that arise, will have an organized, rich, and lively learning experience.

Optional APs provide enrichment and further experiences which can be used if time permits.

Student information APs are intended as reference reading for students but are usually not assigned or collected.

We mention two additional features of each chapter. First, the last AP for each chapter is titled “How Do I Know if I Understand.” This student information AP summarizes the central ideas and main types of problems covered in the chapter.

A second feature is that most chapters include two interchangeable APs, Version A and Version B, of a set of summary problems based on the core ideas of the chapter. In the tables that follow, Version A is identified as core and Version B as student information; however, these roles could be switched. Another possible use is to assign one version when a chapter has been covered and then use the other version later as a review in preparation for an hour exam or final.

Chapter 1: Polygon and Angle Relationships

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 1.0	Introduction 15
AP 1.1	Parallel Line Grid: Sum of the Angles of a Triangle		17
	AP 1.2	Envelope Fold: Sum of the Angles of a Triangle	19
AP 1.3	Justifying the Sum of the Angles of a Triangle or Quadrilateral by Tearing		21
AP 1.4	Sum of the Angles of Any Polygon: How Many Triangles?		23
AP 1.5	The Angles of a Polygon		25
	AP 1.6	When Does Erika's Idea Work?	26
	AP 1.7	The Greedy Triangle	27
AP 1.8	Problems: Sums and Relationships of Angles		29
AP 1.9	Four Kinds of Related Angles		31
	AP 1.10	Figuring Angles and Checking by Measurement	33
AP 1.11	Parallel Lines: How to Recognize Them		35
	AP 1.12	Measuring Sides and Angles of Triangles	39
AP 1.13	Convex: Different Ways to Make Sense of It		41
AP 1.14a	Angle Problems: Version A		43
		AP 1.14b	Angle Problems: Version B 45
	AP 1.15	More Angle Problems	47
		AP 1.16	How Do I Know if I Understand? 49

Addendum to Chapter 1: Thinking Processes: Observing, Reflecting, and Making Sense

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
AP 1.17	Conjecturing about Quadrilaterals		53
AP 1.18	Possible or Not?		54
AP 1.19	True or False (with Example)		55
AP 1.20	Under What Conditions?		56

Chapter 2: Quadrilaterals and Their Definitions

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 2.0	Introduction 57
AP 2.1		Checking Properties of Quadrilaterals	59
AP 2.2		Properties of Quadrilaterals	63
AP 2.3		Marking Quadrilateral Properties	65
AP 2.4		Properties of Diagonal of Quadrilaterals	67
	AP 2.5	Checking Quadrilaterals by Folding	69
	AP 2.6	Read Carefully: Every Word Counts!	73
AP 2.7		Checking Examples Visually or Physically	75
	AP 2.8	Exploring Medial Quadrilaterals	77
AP 2.9a		Problems: Properties of Quadrilaterals, Version A	83
		AP 2.9b	Problems: Properties of Quadrilaterals, Version B 85
	AP 2.10	More Problems: Properties of Quadrilaterals	87
<i>Additional activities with definitions (Optional)</i>			
	AP 2.11	A Deeper Understanding of Definitions	91
	AP 2.12	Special Cases of Quadrilaterals	93
	AP 2.13	Definitions: Inclusive or Exclusive	95
	AP 2.14	Problems: Inclusive and Exclusive Definitions	97
	AP 2.15	What is a Kite? Equivalent Definitions	99
	AP 2.16a	Problems: Definitions of Quadrilaterals, Version A	101
	AP 2.16b	Problems: Definitions of Quadrilaterals, Version B	103
	AP 2.17	More Problems: Definitions of Quadrilaterals	105
		AP 2.18	How Do I Know if I Understand? 107

Chapter 3: Constructions by Paper Folding

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 3.0	Introduction 112
AP 3.1	Introducing CDs: Two Basic Constructions		113
AP 3.2	CD Problem: A Parallel Line		115
	AP 3.3	CD Problem: The Median	116
AP 3.4	CD Problem: An Equilateral Triangle		117
AP 3.5	CD Problem: A Square		118
AP 3.6	Circumscribing Circle		119
AP 3.7	Inscribed Circle		120
AP 3.8	Balance Point of a Triangle		123
AP 3.9	Additional CD Problems Using Basic Construction Steps		124
	AP 3.10	Group Problem: Inscribed Circles	125
	AP 3.11	Folding a Six-Pointed Star or "Snowflake"	127
	AP 3.12	Problems Involving Paper Folding	129
	AP 3.13	How Do I Know if I Understand?	131

Chapter 4: Explorations in Three-Dimensional Geometry

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 4.0	Introduction 133
AP 4.1	Polyhedra (Solids) from and Envelope		137
	AP 4.2	Roll-and-Fold Prism and Pyramid Activities	139
AP 4.3	Net Project A: Prisms		143
AP 4.4	Prisms		151
AP 4.5	Making Sense of Volume: A Basic Relationship		153
AP 4.6	Net Project B: Pyramids		155
AP 4.7	Pyramids		159
AP 4.8	Edges, Faces, and Vertices of Polyhedra		161
	AP 4.9	Special Kinds of Polyhedra	163
	AP 4.10	Riddles with Solids	165
AP 4.11	Volumes of Prisms, Pyramids and Spheres		167
	AP 4.12	Volume of a Pyramid	169
	AP 4.13	What Does Volume Really Mean?	171
AP 4.14	Volume of Solids: First Try		175
AP 4.15a	Solid-Geometry Problems: Version A		179
		AP 4.15b	Solid-Geometry Problems: Version B 181
	AP 4.16	More Solid-Geometry Problems	183
<i>Addendum: Unit Origami: An Introduction</i>			187
	AP 4.17	Instructions for the Basic Parallelogram Unit	189
	AP 4.18	Project for the Whole Class: Monster Stellated Icosahedron	191
	AP 4.19	Unit Origami Projects	193
	AP 4.20	Some Geometry of Unit Origami	195
	AP 4.21	Convex Deltahedra: How Many Are There?	197
	AP 4.22	Problems: Unit Origami	199
		AP 4.23	How Do I Know if I Understand? 201

Chapter 5: Area

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 5.0	Introduction 205
AP 5.1			How Much Space in a Triangle? 209
AP 5.2			Areas on a Geoboard 211
AP 5.3			Two Ways: Cut-up and Take-away 213
AP 5.4			Areas: Parallelograms and Trapezoids 215
AP 5.5			Area by Julie's Way 217
AP 5.6			Which Ways Work for these Figures? 219
AP 5.7			Areas: How Many Ways? 221
AP 5.8			Area Problems: First Try 223
AP 5.9			A Sampling of Area Problems 225
		AP 5.10	Making Sense of Common Units for Length and Area 227
AP 5.11a			Area Problems: Version A 229
		AP 5.11b	Area Problems: Version B 231
		AP 5.12	More Area Problems 233
		AP 5.13	How Do I Know if I Understand? 237

Chapter 6: Explorations with Geoboard Areas

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 6.0	Introduction 239
AP 6.1			Areas of Skew Quadrilaterals 241
AP 6.2			Solid Tile Shapes 243
AP 6.3			Problems: Tile Shapes 245
AP 6.4			Areas of Tile Shapes 247
AP 6.5			Areas by Counting Pegs 249
		AP 6.6	How Many Tile Shapes with Five Squares? 251
AP 6.7			Counting Areas: Pick's Formula 253
AP 6.8			Skew Figures 255
AP 6.9			Discovering, Describing and Using Relationships 257
		AP 6.10	Sean's Idea: Area = Inside Pegs 259
AP 6.11a			Problems: Geoboard Areas, Version A
		AP 6.11b	Problems: Geoboard Areas, Version B
		AP 6.12	More Problems: Geoboard Areas 265
		AP 6.13	How Do I Know if I Understand? 269

Chapter 7: Similarity and Slope

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 7.0	Introduction 271
AP 7.1		Slope or Steepness	273
AP 7.2		Slope: Parallel and Perpendicular	275
AP 7.3		Slope Problems, Part 1	277
AP 7.4		Slope Problems, Part 2	279
	AP 7.5	Linear Equations, Tables of Values, and Slopes	281
AP 7.6		Similar Figures and Their Properties	283
AP 7.7		Similar Figures and Proportionality	285
	AP 7.8	Measuring Proportionality	287
	AP 7.9	Reasoning with Similar Triangles	289
AP 7.10		Similarity and Scale Factors (Length Factors)	291
AP 7.11		Scaling, Areas, and Area Factors	293
AP 7.12		Scaling Problems, First Try	295
AP 7.13		Scaling Problems	297
	AP 7.14	Scaling and Volume of Solids	299
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		AP 7.15b Problems: Slope, Similarity, and Scaling, Version B	303
	AP 7.16	More Problems on Slope, Similarity, and Scaling	305
		AP 7.17 How Do I Know if I Understand?	313

Chapter 8: Pythagorean Theorem and Perimeter

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 8.0	Introduction 317
AP 8.1		Right Triangles of Squares	319
	AP 8.2	Pythagorean Puzzles	321
AP 8.3		Estimating Perimeters on a Geoboard	325
AP 8.4		Slant Lengths on a Geoboard	327
AP 8.5		Geoboard Perimeters	329
AP 8.6		Three Special Triangles	331
AP 8.7		Pythagorean Problems, First Try	333
AP 8.8a		Perimeter and Right Triangle Problems, Version A	335
		AP 8.8b Perimeter and Right Triangle Problems, Version B	336
	AP 8.9	More Perimeter and Right Triangle Problems	337
		AP 8.10 How Do I Know if I Understand?	345

Chapter 9: Geometry of Circles

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 9.0	Introduction 347
AP 9.1		Perimeter (Circumference) of a Circle	351
	AP 9.2	Area of a Circle	353
AP 9.3		Area and Perimeter of Circles and Sectors	355
AP 9.4		Area Problems with Circles, First Try	357
AP 9.5		Problems: Area and Perimeter of Circles	359
	AP 9.6	Inscribed Angles or Arcs of Circles	361
	AP 9.7	The Law of Thales	363
	AP 9.8	Circumscribed or Cyclic Polygons	365
	AP 9.9	Circumscribing Circle for a Cyclic Quadrilateral	367
	AP 9.10	Problems: Inscribed Angles and Circumscribed Polygons	369
AP 9.11a		Problems: Geometry of Circles, Version A	371
		AP 9.11b	Problems: Geometry of Circles, Version B 373
	AP 9.12	Revisiting Volumes: Cones and Cylinders	375
	AP 9.13	Surface Area of an Orange	377
	AP 9.14	More Problems: Geometry of Circles	379
		AP 9.15	How Do I Know if I Understand? 385

Chapter 10: Straightedge and Compass Constructions

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 10.0	Introduction 389
AP 10.1		Basic Straightedge and Compass Constructions	391
AP 10.2		Straightedge and Compass: Construct a Parallel Line	393
AP 10.3		Examples: Reasoning in Construction Problems	395
AP 10.4		Reasoning in Construction Problems	397
<i>(The following are not optional if Chapter 11 will be covered.)</i>			
	AP 10.5	Making Triangles, I: Side-Side-Side	399
	AP 10.6	Making Triangles, II: Side-Angle-Side	401
	AP 10.7	Making Triangles, III: Angle-Side-Angle	403
	AP 10.8	Making Triangles, IV: Side-Side-Angle (Ambiguous Case)	405
	AP 10.9	Congruence Conditions for Triangles	407
		AP 10.10	How Do I Know if I Understand? 409

Chapter 11: Congruence Conditions and Reasoning from Definitions to Properties

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 11.0	Introduction 411
AP 11.1			Congruence Conditions for Triangles and CPCT 413
AP 11.2			Problems: Congruence Conditions and CPCT 415
AP 11.3			Justifications by Congruence Conditions 417
AP 11.4a			Problems: Congruence Conditions, Version A 419
		AP 11.4b	Problems: Congruence Conditions, Version B 421
	AP 11.5		More Problems: Congruence Conditions 423
AP 11.6			From Definitions to Properties: Five-Step Reasoning 425
AP 11.7			Example: Five-Step Reasoning, Problem A 427
AP 11.8			Five-Step Reasoning, First Try 429
AP 11.9			More Problems Using Five-Step Reasoning 431
		AP 11.10	How Do I Know if I Understand? 433

Chapter 12: Computer Constructions

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 12.0	Introduction 437
AP 12.1			Getting Started with Computer Construction Software 438
AP 12.2			Constructing Objects: Midpoints 439
AP 12.3			Constructing Objects: Bisectors 441
AP 12.4			Constructing Objects: Altitudes and Medians 443
	AP 12.5		The Euler Line of a Triangle 445
	AP 12.6		The Nine-Point Circle of a Triangle 447
	AP 12.7		The Medial Quadrilateral of a Quadrilateral 449
AP 12.8			Problems: Investigating Relationships by Using Geometric Properties 451
		AP 12.9	How Do I Know if I Understand 453

Chapter 13: Computer Explorations

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 13.0	Introduction 455
AP 13.1		Triangle Inequalities	457
AP 13.2		Angle Bisectors: Why the Incenter Works	459
AP 13.3		Perpendicular Bisectors: Why the Circumcenter Works	461
AP 13.4		Medians and the Centroid of a Triangle	463
	AP 13.5	Altitudes: The Orthic Triangle	465
	AP 13.6	Angle Bisectors, Medians, and Altitudes: Some Relationships	467
AP 13.7		Revisiting the Medial Triangle: Perimeter and Area	469
	AP 13.8	Revisiting the Medial Quadrilateral: Area	471
	AP 13.9	Quadrilaterals and Circles	473
AP 13.10		Circles: Central Angles and Inscribed Angles	475
AP 13.11		Circles: More on Inscribed Angles and Arcs	477
AP 13.12		Problems: Investigating Relationships by Using Number Ideas	479
		AP 13.13	How Do I Know if I Understand? 481

PART 5: Mira (Reflecta) and Tracing Paper

Chapter 14: Mira Constructions

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 14.0	Introduction 485
AP 14.1		The Mira: What Does it Do?	487
AP 14.2		Reflection Lines and Point-Image Segments	489
AP 14.3		Constructions with a Mira (CDs)	491
	<i>This chapter is completed by assigning and discussing several CD problems from Appendix 3 to be done with a mira.</i>		
	<i>The remaining 5 APs are secondary. Four deal with altitudes. The last is an investigation also examined in AP 3.6.</i>		
	AP 14.4	Altitudes of a Triangle	493
	AP 14.5	Altitudes, Orthocenters, and Trapezoids	495
	AP 14.6	Altitude Constructions with a Mira	497
	AP 14.7	Measuring a Triangle's Three Altitudes	499
	AP 14.8	Where is the Circumcenter?	501
		AP 14.9	How Do I Know if I Understand? 503

Chapter 15: Symmetry

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 15.0	Introduction 505
AP 15.1	Miniproject: Fold-and-Cut Paper Figures		509
AP 15.2	Fold-and-Cut (Symmetric) Shapes		511
AP 15.3	Orientation: One or Two Sides?		513
AP 15.4a	Problems: Symmetry, Version A		515
		AP 15.4b	Problems: Symmetry, Version B 516
	AP 15.5	Fold and Cut: Three Symmetry Lines 517	
	AP 15.6	Fold and Cut: Fivefold Symmetry 519	
	AP 15.7	Problems: More on Symmetry 521	
		AP 15.8	How Do I Know if I Understand? 525

Chapter 16: The Four Symmetries

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 16.0	Introduction 527
AP 16.1	Four Actions: Slide, Flip, Turn, and Glide-Flip		531
AP 16.2	Four Symmetries		533
	AP 16.3	Translations and Coordinates 535	
AP 16.4	Problems: Four Actions or Symmetries		537
	AP 16.5	Combinations of Reflections 539	
AP 16.6	Actions: Which of the Four Types?		541
AP 16.7	Rotations and Glide Reflections: Point-Image Segments		543
AP 16.8	How Do You Get from One to the Other?		545
AP 16.9	CD Problem: Find the Center of Rotation		547
AP 16.10	CD Problem: Find the Glide-Reflection Line		549
	AP 16.11	An Experiment with the Four Kinds Principle 551	
	AP 16.12	Marking Symmetries on Wallpaper Designs 555	
AP 16.13a	Problems: Four Types of Symmetry, Version A		557
		AP 16.13b	Problems: Four Types of Symmetry, Version B 559
	AP 16.14	More Problems Involving the Four Types of Symmetry 561	
		AP 16.15	How Do I Know if I Understand? 567

Prologue: Symmetries of Decorative Art

Chapter 17: Symmetries of Mandalas

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 17.0	Introduction 571
AP 17.1	Symmetries of Mandalas		573
AP 17.2	Classifying Mandalas, First Try		575
AP 17.3	Classifying Mandalas		577
	AP 17.4	Mandalas: One or Two Sides?	579
		AP 17.5	Template Design Mandalas 581
	AP 17.6	Template Design Problems	583
	AP 17.7	Express Yourself with a Mandala	585
		AP 17.8	The Symmetry Classification of Mandalas 587
		AP 17.9	Problems: Mandalas 588
AP 17.10a	Problems: Mandalas, Version A		589
		AP 17.10b	Problems: Mandalas, Version B 591
		AP 17.11	How Do I Know if I Understand? 593

Chapter 18: Symmetries of Borders

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 18.0	Introduction 595
AP 18.1	Glide-Reflection and Half-turn Symmetry		596
AP 18.2	Classifying Borders, First Try		597
AP 18.3	Borders: What Is Their Symmetry Type?		599
	AP 18.4	Generating Borders	601
	AP 18.5	Borders: Make Your Own Display	602
		AP 18.6	The Symmetry Classification of Borders 603
		AP 18.7	Problems: Classifying Borders 607
AP 18.8a	Problems: Borders, Version A		609
		AP 18.8b	Problems: Borders, Version B 611
		AP 18.9	How Do I Know if I Understand? 613

Chapter 19: Escher-Style Tessellations

Core Activities ↓	Additional Activities ↓	Info. for Students ↓	
		AP 19.0	Introduction 615
AP 19.1		Escher Tessellations, Type TTTT	617
		AP 19.2	How to Make a Type TTTT Tessellation 619
AP 19.3		Cut and Tape: Make Your Own Tessellating Shape	620
		AP 19.4	Miniproject: Recognizability 621
AP 19.5		Four Moves for Tessellating Squares	623
AP 19.6		What Are the Possible Heesch Types?	625
AP 19.7		What Is the Heesch Type?	627
		AP 19.8	Project: Making Escher-Style Tessellations 629
AP 19.9		Checking Understanding of Heesch Types	631
		AP 19.10	Marking Symmetries on Escher Tessellations 633
		AP 19.11	Do These Tessellations Work? 635
		AP 19.12	How Do I Know if I Understand? 637