Basic Guidelines for Course Management

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Introduction

Leafing through the Geometric Structures text, it clearly is not a typical college mathematics textbook. This document introduces the features and teaching practices that go with this text especially those which may not be common. We want to be brief providing just enough to get you started with your class.

Activity pages (APs)

This text consists entirely of single-sheet or, in a few cases, multiple-sheet “activity pages” or APs. Each day three to five APs can be assigned for the next class. Alternately, one or more can be worked as a group activity during part of a class. Each AP is a mix of tasks serving one or more of the following purposes:

(a) New ideas: introducing and illustrating new ideas or situations,
(b) Motivating ideas: raising questions and stimulating discussion and awareness of important issues,
(c) Revisiting ideas: revisiting a topic introduced earlier (often in a different context), or
(d) Practicing ideas: using or applying an idea introduced in an earlier AP.
After being introduced in an opening AP, important concepts or especially difficult ideas will be revisited and used in a succession of APs. Class discussions of opening APs provide insight and guidance when these topics come up again later.

**APs are “student tries first”**

The activities in this text are intended to be “student tries first.” This means that APs are assigned and students need to try to do them before they are discussed in class. This contrasts sharply with the more typical “template problem approach” to teaching mathematics where the teacher does several template or model problems in class and then assigns homework very much like the model problems.

The focus of the class’ inquiry is provided by the common experience of having read, attempted and reflected on the activities in advance of class. This common experience forms a basis for class and group discussions. Often times students will not fully understand what is presented; however, this is just fine since it still sets the stage for making sense of the class discussions on the activity.

It is very important to remind students, especially during the first few weeks of class, of the importance of attempting the APs in advance of class. This takes some adjustment on the part of both teachers and students; however, although not perfectly, it does work!

**No lecturing needed**

These materials were developed very explicitly with the intention that traditional lecturing is not necessary. When a student works out an understanding either by recall, reflection, or reading or by engaging in discussion with peers we believe that more substantial and long-term learning takes place\(^1\). Important points of fact and understanding are addressed in a timely and natural way through problems, activities, examples and text passages in the APs. Small group or whole class discussions in response to problems encountered in APs provide an engaging and lively alternative to lectures.

We expect students to take a more active and involved role in their learning. With this approach we, as teachers, also must stretch ourselves with new roles. It is difficult for many of us to restrain from explaining everything. We encourage you to work to reduce class time spent on lecturing. By managing the discussion and group work stimulated by doing assigned APs, we can support our students as they work toward their own individual understanding of geometry. A primary goal of this approach is to support students’ growth as autonomous learners.

\(^1\) Much research supports the view that more learning for long-term retention occurs when students practice recall of knowledge rather than practice knowledge recalled for them by the teacher through lectures. For a short and interesting overview of this issue see “Applying the Science of Learning to the University and Beyond, Teaching for Long-term Retention and Transfer,” by Halpern and Hakel in *Change*, pages 36-41, July/August 2003.
Student and teacher support for this approach

The PDF document *Transparencies for Class Orientation* on the instructors’ website, which includes transparencies and instructions for their use, can be used during the first few classes to bring attention to aspects of the approach which may be new for students (and teachers). These transparencies are based on the excerpt *Major Shifts in Teaching*, taken from the NCTM Professional Standards, which is reproduced on pages xvi and xvii at the front of the text. Topics include:

- Helping students to work together to make sense of mathematics
- Helping students to rely more on themselves in determining whether something is mathematically correct
- Helping students learn to reason mathematically
- Helping students learn to conjecture, invent, and solve problems
- Helping students to connect mathematics, its ideas, and its applications.

We also suggest occasionally mentioning to your class the Student Guide: *Making Sense of Geometry in an Inquiry Class* (pages 641-650) at the back of the Geometric Structures text as well as the *Skills Chart* on page xv.

Concurrent coverage of two or more chapters

A particularity of using this text which we have found beneficial is to assign activities from two or more chapters at the same time, a practice we call concurrent coverage of topics. We believe that it is important to develop the thread of each chapter at its own natural pace. Experiences with new ideas need to be presented, discussed, used, and allowed to settle before additional dependent ideas are presented.

Here is how this can work. A typical chapter may have 10 core APs covering the main ideas which could be covered in about three classes (typically an AP can be covered with 10 to 15 minutes of class time). However, we believe that allowing ideas to develop naturally requires that the material be stretched out over five or six classes with just one or two APs assigned on the topic per class. This means that only 20 or 30 minutes of time per class are needed for the treatment of one chapter’s ideas. In the meantime, another chapter is covered concurrently.
This concurrent coverage allows a more natural pace of making sense of a chain of dependent ideas while still covering substantial amounts of material. The PDF document *Detailed Syllabus with Concurrent Coverage of Chapters 1, 2, and 3* in the instructors’ website shows one possible order of coverage for the course.

**Different types of APs (page icons)**

The inside front cover of the text identifies the eight types of activities with corresponding page icons used in the text. We briefly mention four types (daily activities, problem activities, group activities, and first-try activities) whose icons, respectively, are as follows.

![Daily activities](image1) ![Problem APs](image2) ![Group APs](image3) ![First Try APs](image4)

*Daily activities*, with the sun icon, are the most common APs. These provide a coherent series of tasks and problem situations to give direction to the development of topics. *Problem activities*, with the question mark icon, ask students to make sense of fairly typical geometry problems which are usually introduced and developed in earlier daily activities. Both these types of activities are designed (and field tested) to be appropriate for this audience to do as homework (working individually or with classmates) without advance commentary by instructors.

As a rule, *group activities*, with the cluster of people icon, deal with slightly more difficult topics which, for this audience, are appropriate and manageable when done with the support of a group (but may not be successfully done by many students working individually). These are intended to be used in class. Typically 10 to 15 minutes is ample time for most groups to make sense of these group activities. After this initial discussion as part of a group, students can then complete the activity as homework to be turned in at the next class.

*First-try activities*, with the sunrise icon, can be assigned either as individual homework or as group in-class tasks. These APs serve as an introduction to important, related “families” of problems such as, for example, applied Pythagorean Theorem problems (AP 8.7) or area of compound circular regions problems (AP 9.4). In first-try activities, two or three model problems representing a family are given with no teacher introduction. The discussion of attempts on these problems during the next class then helps more students make sense of additional problems of this type contained in later problem APs. With this approach students are more actively engaged in the process of mastering standard types of problems compared to their passive role in the traditional approach where the teacher lectures on model problems and then assigns more of the same.
Please add your own activities

Faculty teaching courses for perspective teachers are wonderfully creative teachers who often supplement their courses with their own favorite activities and projects. We hope that the modular structure and range of activities in our text welcomes and even stimulates teachers to develop and incorporate their own supplementary activities. The enthusiasm we have for our own activities coupled with the spontaneity provided by active student involvement can give an immediacy and vitality to our classes.

The authors would like to know about activities used as supplements to the Geometric Structures text. With your permission, these will be shared with the family of users of the text via the teachers’ website.

Typical Daily Routine

One possible model for a general structure for class meetings will be outlined here. This model breaks a typical class into two major blocks of time as shown in the next table.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Class Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discussion of assigned APs</td>
<td>60% -- 80%</td>
</tr>
<tr>
<td>Group and miscellaneous activities</td>
<td>20% -- 40%</td>
</tr>
<tr>
<td>Announce next assignment</td>
<td>Brief</td>
</tr>
</tbody>
</table>

As a rule, the largest block of class time is spent discussing assigned activities. With experience we have found that this can be accomplished with little lecturing and in ways that engage and actively involve students. Patience is needed here; awkward moments of silence are to be expected. We list a few of the prompts teachers have found useful for making the transition to a more active role by students.

<table>
<thead>
<tr>
<th>Teacher Prompts to Stimulate Student Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let’s look at AP . . . Does anybody have a question on this page?</td>
</tr>
<tr>
<td>Which problems did anybody have trouble with?</td>
</tr>
<tr>
<td>Are there any problems that somebody would like to have explained?</td>
</tr>
<tr>
<td>Will somebody show us what they did on problem n?</td>
</tr>
<tr>
<td>I want somebody from group (identify the group) to show us the group’s ideas on such-and-such (problem or issue).</td>
</tr>
<tr>
<td>Any thoughts on the way Amy did this problem?</td>
</tr>
<tr>
<td>Did anybody do this a different way?</td>
</tr>
</tbody>
</table>

Some APs will go smoothly; others are more troublesome. With experience teachers will learn what to expect. The document “Chapter-by-Chapter Guide for Teachers” identifies issues in different chapters and in specific APs. Of course, surprises are bound to occur; this is one aspect that makes teaching such a wonderful experience for both teacher and student.
The second major block of time is identified in the table above as “Group and miscellaneous activities.” While the content and time spent on this block will vary considerable, the most common activities are either (a) assigning an AP from the text for students to work in groups or else, (b) having students, again working in groups, begin work on an AP in the next assignment. APs used in this block of type (a) are often chosen from the “optional” activities (see below for mention of optional versus core activities). If the next assignment includes an AP covering more difficult ideas, using class time for students to work on it in groups can help students make sense of these ideas.

**Construct/Describe (CD) problems**

Construct/Describe problems (CD Problems), a central feature of the text, are a variation of traditional straightedge and compass constructions. We briefly describe some aspects of this type of problem and how they can be used in the course. Also see pages 109 through 111 of the text for a discussion of CD problems. The page icon for APs with construction problems pictures a hard hat and a blueprint scroll.

These problems are explored in four contexts.

<table>
<thead>
<tr>
<th>Context</th>
<th>Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Folding</td>
<td>Chapter 3</td>
</tr>
<tr>
<td>Straightedge and Compass</td>
<td>Chapter 10</td>
</tr>
<tr>
<td>Computer Software</td>
<td>Chapter 12</td>
</tr>
<tr>
<td>Mira or Reflecta</td>
<td>Chapter 14</td>
</tr>
</tbody>
</table>

In each context, basic constructions (perpendicular bisector, angle bisector and equilateral triangle) are followed by more involved constructions, such as, for example, constructing the circumscribing circle or the balance point for a triangle. Executing the construction in the appropriate context (for example, making an equilateral triangle with given side length by paper folding) is only the first half of a Construct/Describe problem. Equally important is the describe part: a step-by-step description of the procedure used for the construction.

Following a brief 15 to 20 minute introduction to the basic constructions in each context, we normally will assign one or two CD problems each week. Asking one or two students to read the description part of their completed CD problem often stimulates lively discussions of both mathematical questions and of writing or communication issues. This takes about 10 minutes of class time per assigned CD problem. In this way, through a series of brief, but regular, discussions of constructions in two or three different contexts over the course of a semester, we believe that students can make substantial
improvement in their understanding of constructions and in their ability to describe geometric ideas.

CD problems can be done concurrently at the same time as other topics and chapters. We normally introduce constructions by paper folding beginning with AP 3.1 in the first or second week. One approach is to introduce a new context after each hour exam break. Note that, while CD problems using paper folding are presented in Chapter 3 on separate activity pages (AP 3.1 through AP 3.8), in later contexts only the basic constructions are presented on separate APs. Additional CD problems from all contexts can be assigned out of Appendix 3.3 Catalogue of CD Problems (See page 654). We note that Appendix 3 (pages 652-657 of the text) has additional information about CD problems.

We briefly comment on some of the considerations behind our adoption of this approach to constructions. First of all practice at writing is provided which helps develop both the communication skills so important for teaching and a level of reflection and awareness important for learning. Concrete contexts provide a physical setting to ground geometric thinking and promote group discussion. As more advanced constructions are revisited in a second or third context, higher levels of success foster a sense of achievement and growth. Finally, we feel this approach can be adapted to younger school students and thus can provide a solid foundation for their success with the straightedge and compass constructions of high school.

**Core APs and optional APs**

The document *Chapter Outlines: Core and Optional APs* identifies the *core activities* for each chapter which form a coherent thread or story running through the central ideas of each chapter. This document also identifies *optional activities*, those activities that provide enrichment and depth to the story, but are not essential to the development of the central ideas of the chapter.

A second resource for seeing the central core of each chapter is provide by the final AP in each chapter titled “How Do I Know if I Understand?” These APs, intended as an aid to students as they review, include a listing of the *Basic Relationships (Big Ideas)* and *Basic Types of Problems* for each chapter.

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2 In her 2004 doctoral dissertation at Oklahoma State University, titled *Impact of a Non-traditional Geometry Course on a Prospective Elementary Teachers’ Attitudes and Teaching Efficacy*, Julian Utley writes (page 78) “On the post-questionnaire, prospective elementary teachers were asked what part of the course had impacted them the most as future teachers. More than half of all participant responses indicated that CD problems had impacted them the most. Examination of the responses revealed that participants believed that CD problems improved their mathematical thinking and their ability to communicate that thinking.” She goes on to write (page 79) “Analysis of interview data revealed that participants found CD problems to be frustrating, valuable, interesting, and increased understanding.”
Emphasis on writing, describing and conjecturing

A commonly held belief among students is that answers to math questions are always numbers or calculations; writing is not supposed to happen in math classes. In an attempt to encourage students break away from this narrow view and to reflect and write out what they are doing and thinking, APs are sprinkled with the following types of prompts:

<table>
<thead>
<tr>
<th>Your Response:</th>
<th>Your Reasoning:</th>
<th>Your comments:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your Description:</td>
<td>Your Observations:</td>
<td>Your Conjectures:</td>
</tr>
<tr>
<td>Your Groups’ Reasoning:</td>
<td>Your Work:</td>
<td></td>
</tr>
</tbody>
</table>

We believe that it is important to regularly call on students to share what they have written for these prompts and to make note when correcting homework if these prompts are answered fully. Paying attention to responses to these prompts helps students gain more confidence in their ability to write and verbally express mathematical ideas, important skills for teachers.

Higher Order Thinking

Four special types of problems and activities are introduced in four APs in an Addendum to Chapter 1 titled “Thinking Processes: Observation, Reflection, and Making Sense” (See pages 51 through 56).

<table>
<thead>
<tr>
<th>Problems Involving Higher Order Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP 1.17 Conjecturing (observing possible relationships in situations)</td>
</tr>
<tr>
<td>AP 1.18 Possible or Not Problems (with supporting examples if possible)</td>
</tr>
<tr>
<td>AP 1.19 True or Not True Problems (with counterexamples if not true)</td>
</tr>
<tr>
<td>AP 1.20 Under What Conditions? Problems (identifying conditions for a statement to be true)</td>
</tr>
</tbody>
</table>

After being introduced at the end of Chapter 1, examples of these types of problems and activities are sprinkled throughout the rest of the text. Regular use of these types of problems in class and on exams supports the development of higher order thinking skills.

Exams

Sample exams are included on the Teacher’s Website in Microsoft Word documents. These can be downloaded and modified to serve your purposes. The student website includes PDF versions of lots of exams which students can print for study or review.
Collecting and grading homework

Several schemes for handling homework collection and grading has been successfully employed with this text. One scheme is to collect and grade all assigned APs. However, due to the volume of paperwork, grading needs to be cursory, often just spot checking or checking off completion. Another scheme is to collect just one AP randomly each day which is then graded more rigorously. Some teachers collect APs before they are discussed; others collect assignments after they are covered in class.

Final note

Many practices not widely used by college teachers are supported in this text such as, among others, strong emphasis on activities and manipulatives, letting students struggle with problems without showing how to do them in advance, constructions in many contexts including miras and paper folding, extensive use of group work, little or no lecturing. We described some of these practices in this document. We (Aichele and Wolfe) do firmly believe that a lively and engaging course that works very well for prospective elementary and middle school teachers can be organized around all of these practices. We also know that too much newness can be overwhelming to the teacher and can produce anxiety among students. We encourage and welcome users of this text to adapt the activities and the teaching practices we describe to their own style of teaching. All of our suggestions are just that – suggestions. We must each develop our own style of teaching that works for us and our students.