Paper Folding:
Two Basic Constructions
And Why They Work
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Perpendicular Bisector of a Line Segment

What does Perpendicular mean?
Two lines that form a right angle (90°) are called perpendicular lines. This is often written as \( \perp \).
For example, if line \( m \) is perpendicular to line \( l \), we would write "\( m \perp l \)".

What is a Bisector?
"Bisect" means to cut into 2 equal parts. So the bisector of a line is a line which divides a line segment into two equal parts.
In the illustration below line \( s \) meets line segment \( PQ \) at point \( R \). We can see that the distance between \( P \) and \( R \) is the same as the distance between \( R \) and \( Q \). Therefore line \( s \) is a bisector of line segment \( PQ \).

Line Segment? What do you mean?
Well, a line really never ends. It extends forever in both directions. A line segment is a part of a line but has defined beginning and end points. We can tell the difference between a line and a line segment because a line has arrows on the ends and a line segment has dots.
So...we combine these to form a **Perpendicular Bisector** of a **Line Segment**.

The **perpendicular bisector** of a **line segment** is a line that cuts a line segment exactly in half. Also, the line and the **line segment** form a right angle.

In this example, line $s$ cuts **line segment** $PQ$ exactly in half at point $R$. Also, line $s$ makes a right angle with **line segment** $PQ$. Hence, line $s$ is a **Perpendicular Bisector** of **Line Segment** $PQ$.

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**Paper folding description of a Perpendicular Bisector of a Line Segment**

1. **Start with line segment** $AB$.
2. **Fold the line segment** on top of itself so that point $A$ meets up exactly with point $B$.
3. **Now make a crease in the paper**. Label this crease as line $s$ (marked here as a dashed line).
4. **Line $s$ is the Perpendicular Bisector** of **Line Segment** $AB$. 
How do we know it is a Perpendicular Bisector?

The easiest way to show that line $s$ really is the Perpendicular Bisector of Line Segment $AB$ is by simply measuring.

1. To see if $s$ is a bisector, measure from $A$ to where $s$ intersects $AB$ and from $B$ to where $s$ intersects $AB$. In our case this distance is 1 inch on each side. That means $s$ is a bisector of line segment $AB$.
2. The last thing we need to check is if $s$ forms a right angle with $AB$. By laying a protractor over this angle, we see that indeed $s$ is perpendicular to $AB$.
3. We conclude that $s$ is perpendicular bisector of line segment $AB$. 
Bisector of an Angle

What is an Angle?

An angle is a figure formed by two line segments that extend from the same point. The point where the line segments meet is called a vertex. In the illustration below, line segments KJ and JL form an angle with vertex J. We call this angle "Angle KJL" and write it as \( \angle KJL \).

\[ \text{K} \]
\[ \text{L} \]
\[ \text{J} \]

What is a Bisector?

A bisector is a line that cuts something into two equal parts. In this case we are talking about an angle, so the bisector goes through the vertex.

So, we combine these terms to get the Bisector of an Angle

The Bisector of an Angle cuts an angle in half. We see in the picture below that line \( m \) is the bisector of \( \angle KJL \). This means that \( \angle KJL \) is split in half by line \( m \) so \( \angle KJR \) is equal to \( \angle RJL \) and \( \angle KJR + \angle RJL = \angle KJL \).

\[ \text{K} \]
\[ \text{R} \]
\[ \text{L} \]
\[ \text{J} \]
Paper folding description of a
Bisector of an Angle

Step 1

1. Start with \( \angle KJL \).
2. Fold line segment KJ on top of JL so that the folded line goes through the point J.
3. Now make a crease in the paper. Label this crease as line \( m \).
4. Line \( m \) is the Bisector of Angle KJL.

Step 2 & Step 3
How do we know it is a Bisector of an Angle?

The easiest way to show that line \( m \) really is the Bisector of Angle \( KJL \) is to measure using a protractor.

1. Measure \( \angle KJR \). We find that \( \angle KJR = 20^\circ \)
2. Measure \( \angle RJL \). We find that \( \angle RJL = 20^\circ \)
3. Thus \( \angle KJR = \angle RJL = 20^\circ \).
4. Now add \( \angle KJR \) and \( \angle RJL \). So \( \angle KJR + \angle RJL = 40^\circ \)
5. Measure \( \angle KJL \). We see that \( \angle KJL = 40^\circ \)
6. Therefore, we have \( \angle KJR + \angle RJL = \angle KJL = 40^\circ \) and \( \angle KJR = \angle RJL = 20^\circ \) so we conclude that \( m \) really is the Bisector of Angle \( KJL \).