Hour Exams and Final
Math 3403 – Summer 2001

John Wolfe

Final ......................... page 2
Mini-Exam III .................. page 7
Exam II .......................... page 10
Exam I .......................... page 15

File: e01sum.tex
1. (16 points) Four different geoboard figures are given below. You are to work out the area of each figure using a different method on each figure. Describe the method you use for each figure. Note: Be sure to illustrate 4 different methods.

(a) Method 1:

(b) Method 2:

(c) Method 3:

(d) Method 4:

2. (8 points) Find the measure of each lettered angle in the following figure, \( l \) and \( m \) are parallel lines. Show your reasoning.

3. (8 points) The base of an 8-foot ladder is placed 3 feet away from the wall.

How far up the wall does the ladder reach?
Express your answer two ways: as a square root and as a decimal.
4. (8 points) Two ways, path 1 and path 2, are shown below for getting from point A to point B. Which one is shorter? (Show your work!)

![Diagram of two paths](image)

6. (5 points) Billy said to his sister “Two lengths are enough information to determine a rhombus.”

She disagreed, “I think you are wrong because at least one angle is needed to fix the shape of the rhombus.”

But, Billy is right. What part of the rhombus must the two lengths represent in order to be a congruence condition?

7. (7 points) Two sets of similar figures are given below. In the blanks provided beside each set, indicate the scale factor and the area factor going from the figure on the left to the larger figure on the right.

![Scale Factor and Area Factor](image)

8. (7 points) John’s gerbil pen comfortably holds 12 gerbils. In order to make more room in the garage, John’s dad has told him that he needs to reduce the dimensions of his pen by one-half. How many gerbils will John’s new pen comfortable hold?
Four Step Problem

9. (A10 points)

**Official Definition:** A parallelogram is a quadrilateral in which opposite sides are equal.

**Property:** For official parallelograms, alternate interior angles for a diagonal are congruent.

*Note:* You only need to show for one diagonal. The same presentation will apply to the other diagonal as well. Alternate interior angles for the diagonal are labeled $x$ and $y$ in the figure below.

![Diagram of a parallelogram with labels $x$ and $y$](image)

---

**Step 1**

---

**Step 2**

---

**Step 3**

---

**Step 4**

---

---

**Four Step Model**

<table>
<thead>
<tr>
<th>Step 1: Mark given information on figure: official definition, constructions, related definitions, earlier results.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2: Draw and identify apparently congruent triangles.</td>
</tr>
<tr>
<td>Step 3: Cite and fully apply CC to triangles.</td>
</tr>
<tr>
<td>Step 4: Apply CPCT for results needed for the property.</td>
</tr>
</tbody>
</table>
10. (10 points) A segment $AB$ is drawn below. Using a straightedge and compass, construct an isosceles right triangle so that $AB$ is the hypotenuse.

First carry out your construction. Then write out a step by step description of the process that you use.
11. (10 points) On the triangle below, use paper folding to find the center of the inscribed circle. Use a compass to draw this circle.

Describe the process that you used to find the inscribed circle.

Describe:
1. (8 points) Using the code table below, identify the symmetry type of the following mandalas.

\[ C_n \] \quad n\text{-fold rotational symmetry (no reflectional symmetry)}

\[ D_n \] \quad reflectional symmetry and \( n\)-fold rotational symmetry

\[ D \text{ or } D_1 \] \quad Bilateral symmetry only

\[ N \text{ or } C_1 \] \quad No symmetry

2. (8 points) Using the code table below, identify the symmetry type of these borders.

3. (3 points) Make an example of a pentagon which has bilateral symmetry.
4. (8 points) Remember the instructions for “True or Not Questions?”

Instructions:
* If the statement is true write TRUE and you are done.
* If the statement is false write FALSE and also draw an example showing that it is not true.

(a) True? Fold & Cut shapes are the same as shapes with bilateral symmetry.

(b) True? It is not possible for a figure to have exactly five lines of symmetry.

(c) True? One-sided figures are the same as symmetric figures.

5. (4 points) Fold a sheet of paper in half. Then make two straight line cuts starting and ending on the foldline as pictured. (There are many ways to do these two cuts.)

After unfolding, what kind of shapes will you always have?

6. (6 points) For each of the following pairs of congruent figures indicate if they are related by translation, rotation, reflection or glide reflection.

7. (8 points) For each of the Escher style prototiles given below, identify the Heesch type.
8. (10 points) Notice that the two figures given below have the same orientation. Therefore they must be related by a rotation (since translation does not seem to work). **Using a mira**, find the center of rotation which takes one of the figures to the other.

**Describe** the process that you used in this problem.
1. (8 points) Using the dot paper below, draw and label
   a) a segment with slope \(-\frac{3}{4}\).
   b) a segment with slope 25%.
   c) a segment with slope 6.

2. (3 points) A line is drawn below. Will the line which is
   perpendicular to this line and passes through the point
   \(A\) also pass through point \(a, b, c, d, e, \text{ or} f\)? Circle your
   choice.

3. (9 points) Figure the perimeter of this figure. Express
   your answer with square roots and as a decimal.

4. (8 points) For the following two right triangles, figure
   out the length of the side marked with a question mark.
   Show work!

5. (7 points)
   Mark and Mary are flying a kite. Mark has let out 120
   feet of string and the kite is directly above Mary's head.
   Mary is 90 feet away from Mark.

   How high is the kite over Mary's head?

   Express your answer two ways: as a square root and as
   a decimal.

6. (7 points) Max is planning to enlarge his square cattle
   pasture so that the new fence encloses two times the
   area of the present pasture. He wants the pasture to
   remain square. The original fence is 200 feet long on
   each side. How long should one side of the new fence be?
7. (8 points) For each of the figures given below, write \( CC \) (for congruence condition) if enough information is given to determine the rest of the figure. Write \( not \ CC \) otherwise.

![Kite](image)

![Rhombus](image)

8. (10 points) Remember the instructions for “True or Not Questions?”

Instructions:
* If the statement is true write \( TRUE \) and you are done.
* If the statement is false write \( FALSE \) and also draw an example showing that it is not true.

(a) \( True? \) Any two rectangles are similar.

(b) \( False? \) Knowing the length of one side of a rhombus is a congruence condition (i.e., enough information to determine the figure).

(c) \( True? \) In a right triangle each leg is shorter than the hypotenuse.

9. (6 points) In the triangle below, draw and clearly label each of the three altitudes.

![Triangle](image)
Definitions to Properties – Four Step Model

10. (A12 points)

Official Definition: A rectangle is a quadrilateral with opposite sides the same length and all angles right angles.

Property: For an official rectangle, the two diagonals are the same length (SAS).

Step 1

Step 2

Step 3

Step 4

Four Step Model

Step 1: Mark given information on figure: official definition, constructions, related definitions, earlier results.
Step 2: Draw and identify apparently congruent triangles.
Step 3: Cite and fully apply CC to triangles.
Step 4: Apply CPCT for results needed for the property.
11. (10 points) An angle is drawn below. Using a straight edge and compass, construct the angle bisector of this angle.

First carry out your construction. Then write out a step by step description of the process that you used.
12. (A10 points) Using a mira, locate a point C so that the triangle ABC is equilateral.

Note: Do the construction and then clearly describe the process that you used.

Describe:
1. (10 points) One of the methods we have used for finding areas is:

   \textit{METHOD: base \times height.}

   (a) Make up an figure and illustrate this method below:

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

   (b) Make up a figure where this method does not work.

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

   (c) For what kind of figures does this method work? Describe:

2. (10 points) Use Pick's formula to figure out the area of this figure. \textit{(Show your calculations!)}

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

   On the geoboard below draw a figure for which Pick's formula does not work:

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

3. (7 points) What is the angle sum for the following figure? \textit{Note: Show your reasoning.}

4. (7 points) Find the measure of the angle marked with a question mark. \textit{(Show your work!)}

   \begin{center}
   \begin{tabular}{c}
   \text{Regular Pentagon}
   \end{tabular}
   \end{center}

5. (12 points) Figure \textit{ABCD} is a parallelogram in the picture below. Three of the angles are given.

   \begin{center}
   \begin{tabular}{c}
   \text{ABCD a Parallelogram}
   \end{tabular}
   \end{center}

   (a) What is angle y? \textit{(Show your reasoning.)}

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

   (b) What is angle w? \textit{(Show your reasoning.)}

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

   (c) What is angle x? \textit{(Show your reasoning.)}

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}

   (d) What is angle z? \textit{(Show your reasoning.)}

   \begin{center}
   \begin{tabular}{c}
   \text{.
   
   
   
   
   
   
   }
   \end{tabular}
   \end{center}
6. (6 points)

(a) What is the most general shape which is both a kite and a rectangle?

(b) What is the most general shape which is both an isosceles trapezoid and a parallelogram?

7. (9 points) Diagrams of quadrilaterals are given below. Decide if the information given is possible or not. If it is possible write OK. If there is something wrong, then write not OK and explain what is the matter.

(a)

[Diagram of a parallelogram with angles 120° and 70°]

OK or not: 

Explain if not OK:

(b)

[Diagram of a kite with angles 100°, 70°, 90°, 100°]

OK or not: 

Explain if not OK:

(c)

[Diagram of a parallelogram with angles 120° and 70°]

OK or not: 

Explain if not OK:

8. (6 points) Bill was thinking about GeoZAGs. “I think,” Bill said, “that for a geoZAG the number of edge pegs is always twice the area. The internal pegs do not matter since geoZAGs do not have any.” If you agree with all that Bill said, then write TRUE below. If you do not agree than write NOT TRUE and make up an example on the dot paper below which demonstrates how something that Bill said is wrong.

\[
\begin{array}{cccccccc}
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\end{array}
\]

9. (9 points) True or False (with Example): For each of the following statements either circle TRUE if you believe it is true or, if you believe it is false, circle FALSE and give examples showing why it is false.

(a) Any two equilateral triangles must be congruent.

TRUE

FALSE - these examples show why it is false:

(b) The incenter of a triangle is always inside of the triangle.

TRUE

FALSE - these examples show why it is false:

(c) Any two figures on a geoboard which have area 4 must have the same perimeter.

TRUE

FALSE - these examples show why it is false:
10. (A12 points) On the triangle below, use paper folding to find the center of the **inscribed** circle. Use a compass to draw this circle.

Describe the process that you used to find the inscribed circle.

Describe:
11. (A12 points) Using paper folding, locate a point $C$ so that the triangle $ABC$ is equilateral.

   *Note:* Do the construction and then clearly describe the process that you used.

   Describe: