Hour Exams and Final
Math 3403 – Spring 1999

Version A

John Wolfe

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File: E99s.tex
MATH 3403 – Geometric Structures – Final Exam – A
May 1999

Name: _______________________

1. (A10 points) Using the code table below, identify the symmetry type of these borders.

![Borders](image)

<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
</tr>
<tr>
<td>crossline</td>
<td>centerline sym.</td>
</tr>
<tr>
<td>1</td>
<td>no crossline sym.</td>
</tr>
<tr>
<td>g</td>
<td>glide reflectional sym.</td>
</tr>
<tr>
<td>2</td>
<td>half-turn symmetry</td>
</tr>
<tr>
<td>1</td>
<td>no additional sym.</td>
</tr>
</tbody>
</table>

2. (A8 points) Find the lengths of the unmarked sides of these right triangles. Express your answer two ways: a) as a square root and b) as a decimal.

![Triangles](image)

3. (A15 points) For this problem consider which area methods will work on each of these six figures.

![Figures](image)

Write yes or no in each of the squares in the table below depending on whether the method will work or not work. Look below the table to see a description of the methods.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1: Internal + 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2: Pick’s formula</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3: (Edge/2) minus 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4: (base) * (height)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M5: base * height</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

M1: Count the internal pegs and add one.
M2: Use Pick’s formula or rule.
M3: Count edge pegs and divide by two; then subtract 1.
M4: One half of the base times the height.
M5: The base times the height.

4. (A7 points) Work out the area of this figure. Show your work!

![Figure](image)
5. (A8 points) Pairs of congruent J’s are given below. Indicate which of the four possible relationships (translation, rotation, reflection or glide reflection) holds between the pair.

6. (A8 points) Figure out the perimeter of the following figures. Write your answer two ways: as a sum of square roots and as a decimal.

(a) as a sum of square roots:

(b) as a decimal

7. (A10 points) Three quadrilaterals are given below. Figure the area of each of these quadrilaterals. Next draw the inscribed quadrilateral inside each figure and figure the area of the inscribed quadrilaterals.

Do you notice any relationship between the area of the quadrilateral and the area of the inscribed quadrilateral? Describe:

8. (A5 points) Write in the values of all unlabeled angles and sides on this figure.

Rhombus

5

50°
9. (A9) Two line segments $A$ and $B$ are given on the dot paper below. In addition a dot is circled to indicate a center for a half-turn rotation of $180^\circ$. Draw the image $A'$ of $A$ and the image $B'$ of $B$ under a rotation of $180^\circ$ about the circle point.

Beside each of the four segments write the slope of the segment.

What is the relationship between the slope of a segment and the slope of the image of the segment when rotated through $180^\circ$? Describe the relationship:

10. (A9 points) A pair of parallel lines and a triangle are drawn below. Three of the angles involved are given.

(a) What is angle $a$? Why? (Show calculations.)

(b) What is angle $b$? Why? (Show calculations.)

(c) What is angle $c$? Why? (Show calculations.)

11. (A6 points) The mandala pictured below has symmetry code $D_4$. Note that one angle of $35^\circ$ is marked. Figure out the values of angles $a$, $b$ and $c$. Note: Show your work below.
12. (15 points) Below each of the following tessellations make a copy of its basic shape and indicate its Heesch type.
CD Problems

Warning!! Be sure to use the indicated tool.

Name:__________________

13. (A10 points) Notice that the two figures given below have the same orientation. Therefore they must be related by a rotation (since translation does not seem to work). Using a mira, find the center of rotation which takes one of the figures to the other.

Describe the process that you used in this problem.
14. (10 points) On the triangle below, use a mira to find the center of the inscribed circle. Use a compass to draw this circle.

Describe the process that you used to find the inscribed circle.

Describe:
CD Problem – Straight Edge and Compass

Name:____________________

15. (A10 points) Three lengths or sides are given below. Using a straight edge and compass, make a triangle out of the given information. Begin by copying segment $AB$ onto line $l$ below.

First carry out your construction. Then write out a step by step description of the process that you use.

$A$……………………………………………………………. $B$

$\hspace{2.5\text{in}}$

$\hspace{2.5\text{in}}$
16. (10 points) A segment $AB$ is drawn below. Using paper folding, construct an isosceles right triangle so that $AB$ is the hypotenuse.

First carry out your construction. Then write out a step by step description of the process that you use.
1. (A18 points) Using the code table below, identify the symmetry type of the following mandalas.

![Mandalas](image)

![Code Table](table)

**Code for Mandalas**

<table>
<thead>
<tr>
<th>$C_n$</th>
<th>$D_n$</th>
<th>$D$ or $D_1$</th>
<th>$N$ or $C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-fold rotational symmetry (no reflectional symmetry)</td>
<td>reflectional symmetry and n-fold rotational symmetry</td>
<td>Bilateral symmetry only</td>
<td>No symmetry</td>
</tr>
</tbody>
</table>

2. (A5 points) Imagine that you fold a sheet of paper in half and cut out a figure. This fold and cut shape would be a kind of mandala. What will the symmetry type be of this mandala?

3. (A8 points) Indicate if the following figures are one or two sided by writing a 1 or a 2 beside them.

![Figures](image)

4. (A5 points) A sheet of dotpaper is double folded on the dashed lines as pictured.

![Dotpaper](image)

Imagine that the folded paper is cut along the dark lines indicated above. Draw in the rest of the shape that will result when the paper is unfolded.
5. (A15 points) Using the codetable below, identify the symmetry type of these borders.

[Images of borders a) to e)]

Code for Border Patterns

<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>m crossline sym.</td>
<td>m centerline sym.</td>
</tr>
<tr>
<td>1 no crossline sym.</td>
<td>g glide reflectional sym.</td>
</tr>
<tr>
<td>2 half-turn symmetry</td>
<td></td>
</tr>
<tr>
<td>1 no additional sym.</td>
<td></td>
</tr>
</tbody>
</table>

6. (A5 points) What kind of quadrilaterals (squares, rectangles, parallelograms, rhombuses, kites, trapezoids, and isosceles trapezoids) have bilateral symmetry?

7. (A6 points) In your own words and in terms of using tracing paper, describe how to tell the difference between a border of type 12 and of type 1g.

[Images of borders a) to e) with codes]

8. (A12 points) Pairs of congruent J’s are given below. Indicate which of the four possible relationships (translation, rotation, reflection or glide reflection) holds between the pair:

[Images of J pairs a) to d)]
9. (12 points) Notice that the two figures given below have opposite orientations. Therefore they must be related by a glide reflection (since plane reflection does not seem to work). Using a mira, find the glide reflection line which takes one of the figures to the other.

**Describe** the process that you used in this problem.
10. (A14 points)

(a) A triangle is drawn below with one vertex labeled $A$. Using a mira, draw the altitude to this triangle through the vertex $A$.

Describe the process that you used.

(b) For the triangle given below, draw the altitude which goes through the vertex $B$. 

Description:
1. (A11 points) The dot paper diagram below shows two ways to get from point $A$ to point $B$. Which is shorter, route #1 (solid) or route #2 (dotted)? Show your work.

2. (9 points) On the dot paper below, draw and label a line which
   (a) has a slope of $-\frac{3}{7}$,
   (b) has a slope of .2,
   (c) has a slope of 25%.

3. (A6 points) The area of a square cake pan is 81 square inches. How long is a diagonal for this pan?

4. (A6 points) If you put these three squares together joining corners to surround a triangle, would they form a right triangle of squares?

   How did you come to your conclusion?

5. (A12 points) Four figures are given below with some of their dimensions given. Write CC (for congruence condition) if enough information is given to determine the rest of the figure. Write not CC otherwise.
6. (A10 points) Find the perimeter of this figure. Express your answer as a square root and as a decimal.

![Diagram of a quadrilateral with sides 6, 12, 6, and 6]

7. (A7 points) Two dots are circled in the grid below. Circle two additional dots so that the resulting 4 dots are the corners of a square.

![Grid with two circled dots and some unmarked dots]

Describe clearly how you determined the two dots completing the square:

8. (A7 points) Wanda knows that one ton of sand is needed for her son’s rectangular sandbox. But she is thinking of making each side twice as long. If she does this how much sand will she need?

9. (A6 points) Bill drew a square on a sheet of paper. He was thinking that he wanted a new square whose area was three times as big as the area of the square he already had. By what factor does he have to increase the side length in order to get the area he wants?
10. (13 points) An angle is drawn below. Using a straight edge and compass, construct the angle bisector of this angle.

First carry out your construction. Then write out a step by step description of the process that you used.
11. (13 points) A segment $AB$ is drawn below. Using a straight edge and compass, construct an isosceles right triangle so that $AB$ is one of the legs.

First carry out your construction. Then write out a step by step description of the process that you use.
1. (A25 points) Four different geoboard figures are given below. You are to work out the area of each figure using a different method on each figure. Describe the method you use for each figure. *Note: Be sure to illustrate 5 different methods.*

(a) Method 1:

(b) Method 2:

(c) Method 3:

(d) Method 4:

(e) Method 5:

2. (A5 points) What is the measure of the missing angle?

3. (A5 points) What is the measure of the unknown angle (marked with a ?) in this figure?
4. (A12 points) Diagrams of quadrilaterals are given below. Decide if the information given is possible or not. If it is possible write OK. If there is something wrong, write not OK and explain what is the matter.

(a) (A4 points)

\[\begin{array}{c}
70^\circ & 120^\circ \\
120^\circ & 70^\circ \\
\end{array}\]

Parallelogram

OK or not:

Explain if not OK:

(b) (A4 points)

\[\begin{array}{c}
70^\circ & 90^\circ \\
100^\circ & 100^\circ \\
\end{array}\]

Kite

OK or not:

Explain if not OK:

(c) (A4 points)

\[\begin{array}{c}
70^\circ & 120^\circ \\
120^\circ & 70^\circ \\
\end{array}\]

Parallelogram

OK or not:

Explain if not OK:

5. (A14 points) The seven types of quadrilaterals we have been working with are drawn here.

\[\begin{array}{c}
\text{square} & \text{rhombus} & \text{rectangle} & \text{parallelogram} \\
\text{kite} & \text{trapezoid} & \text{isosceles trap} \\
\end{array}\]

For each of the following descriptions, write down all of the names of the quadrilaterals which satisfy the description. Note: Multiple answers are possible.

(a) A quadrilateral in which there are two pairs of opposite parallel sides.

(b) A quadrilateral in which there is one or more pairs of equal adjacent angles.

(c) A quadrilateral in which the diagonals are perpendicular bisectors of each other.

(d) A quadrilateral in which only two sides are parallel.

6. (A8 points)

(a) What is a shape that is both a kite and a rectangle?

(b) What is a shape that is both a kite and a parallelogram?

7. (A9 points) During our discussions we have seen a type of geoboard figure where we can get the area by using the relationship “inside pegs plus one.” On the first geoboard below draw a typical example where this area relationship holds. On the second geoboard draw an example where it fails to hold.

\[\begin{array}{c}
\begin{array}{c}
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\end{array} & \begin{array}{c}
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet \\
\end{array}
\end{array}\]

Relationship holds. Relationship fails.

Describe as carefully as you can the kinds of figures for which the relationship “Area equals inside pegs plus 1” holds.

Your description:
8. (A-11 points) On the triangle below, use paper folding to find the balance point or center of gravity.

Describe the process that you used to find the balance point.

Describe:
9. (A-11 points) Using paper folding, locate a point $C$ so that the triangle $ABC$ is equilateral.

*Note:* Do the construction and then clearly describe the process that you used.

Describe: