Hour Exams and Final
Math 3403 – Spring 2002

Version A

John Wolfe
1. (A15 points) Using the codetable below, identify the symmetry type of these borders.

![Images of borders a) to e)](image)

**Code for Border Patterns**

<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>m crossline sym.</td>
<td>m centerline sym.</td>
</tr>
<tr>
<td>1 no crossline sym.</td>
<td>g glide reflectional sym.</td>
</tr>
<tr>
<td>2 half-turn symmetry</td>
<td>1 no additional sym.</td>
</tr>
</tbody>
</table>

**Code for Mandalas**

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_n$</td>
<td>n-fold rotational symmetry (no reflectional symmetry)</td>
</tr>
<tr>
<td>$D_n$</td>
<td>reflectional symmetry and n-fold rotational symmetry</td>
</tr>
<tr>
<td>$D$ or $D_1$</td>
<td>Bilateral symmetry only</td>
</tr>
<tr>
<td>$N$ or $C_1$</td>
<td>No symmetry</td>
</tr>
</tbody>
</table>

2. (A3 points) Jane made the template design shown below. She claimed that it was a mandala of type $D_{12}$, but one of her classmates disagreed because some of the arrows are darkened. What is the type of this mandala? The table is given below for you help.

![Mandala with arrows](image)

3. (A9 points) For these two right triangles, figure out the length of the side marked with a question mark. Show your work!

![Triangles](image)
4. (18 points) Listed below are some of the methods we have used to figure areas.

**Cut-up:** Divide the inside of the figure up into smaller areas that are easy to figure. Then add up the smaller areas.

**Take Away:** Enclose the shape in a rectangle. Figure the area of the rectangle. Subtract the area outside the shape (but inside the rectangle).

**Count I:** Count the inside pegs and add 1.

**Count II:** Count the number of edge-peg and divide by 2. Then subtract 1.

**Pick:** Count the number of edge-peg and divide by 2. Then add the number of inside pegs and subtract 1.

Three figures are given below. Under each figure the methods given above are listed. After each method either

(a) write **yes** if the method works for finding the area, or

(b) write **no** and give a reason why the method does not work for finding the area.

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**Figure I:**

![Figure I](image)

**Cut-up:**

**Take away:**

**Count I:**

**Count II:**

**Pick:**

---

**Figure II:**

![Figure II](image)

**Cut-up:**

**Take away:**

**Count I:**

**Count II:**

**Pick:**

---

**Figure III:**

![Figure III](image)

**Cut-up:**

**Take away:**

**Count I:**

**Count II:**

**Pick:**

---

5. (A6 points) The length of one side of a $30^\circ - 60^\circ - 90^\circ$ triangle is given below. Figure out the other two sides. Be sure to show your work!
6. (A8 points) Figure \(ABCD\) is a parallelogram in the picture below. Three of the angles are given.

\[ABCD \text{ a Parallelogram} \]

(\(A\) \(w\) \(x\) \(B\) \(x\) \(w\) \(D\) \(40^\circ\) \(60^\circ\) \(45^\circ\))

(a) What is angle \(x\)? (Show your reasoning.)

(b) What is angle \(y\)? (Show your reasoning.)

(c) What is angle \(w\)? (Show your reasoning.)

(d) What is angle \(z\)? (Show your reasoning.)

7. (A7 points)

The Jones bought a new widescreen high definition TV. The rectangular screen measured 27 inches high by 45 inches wide. However, when TVs are advertised their size is the diagonal length of the screen. What is the advertised size of the Jones’ new TV?

[TV Screen diagram]

8. (A6 points) For this problem you are to name all of the quadrilaterals in which the diagonals bisect each other. Then identify the one which is most general.

Possible names: rectangle, rhombus, parallelogram, square, kite, trapezoid and isosceles trapezoid.

(a) Names:

(b) Which is most general?

9. (A7 points) The area of the second dog drawn below is 3 times as large as the area of the first.

If the small dog’s tail is 2 cm long, how long is the tail of the second dog?

10. (A8 points) Pairs of figures, which look to be congruent, are given below. Additional information about the figures is indicated using the standard markings. After each pair write \(CC\) if there is enough information marked to guarantee the triangles actually are congruent. Write \(Not\ CC\) if there is not enough information marked.

(a) \[\text{Diagram of pairs}\]

(b) \[\text{Diagram of pairs}\]

(c) \[\text{Diagram of pairs}\]

(d) \[\text{Diagram of pairs}\]
11. (A6 points) Below two “bad” definitions for a kite are given. For each of these attempted definitions you are to show how it is bad by drawing and labeling a picture of a quadrilateral which fits the bad definition but is clearly not a kite.

(a) A kite is a quadrilateral that has at least one pair of congruent opposite angles.

Your example:

(b) A kite is a quadrilateral with perpendicular diagonals.

Your example:

12. (A8 points) For each of the Escher style prototiles given below, identify the Heesch type.

13. (A9 points) Analyze each of the Escher style tessellations given below. For each design, indicate the Heesch type.
Four Step Problem

14. (10 points)

**Official Definition:** A parallelogram is a quadrilateral in which opposite sides are equal.

**Property:** For official parallelograms, alternate interior angles for a diagonal are congruent.

*Note:* You only need to show for one diagonal. The same presentation will apply to the other diagonal as well. Alternate interior angles for the diagonal are labeled $x$ and $y$ in the figure below.

```
Step 1

Step 2

Step 3

Step 4
```

**Four Step Model**

Step 1: Mark given information on figure: official definition, constructions, related definitions, earlier results.
Step 2: Draw and identify apparently congruent triangles.
Step 3: Cite and fully apply CC to triangles.
Step 4: Apply CPCT for results needed for the property.
CD Problem – Paper Folding

Name:____________________

15. (A10 points) Using paper folding, find the line which is parallel to the line $l$ and passes through the point $P$.

*Note: Do the construction and then clearly describe the process that you used.*

Describe:
16. (10 points) Using a *mira*, locate the center of the circle passing through J, K, and L. Use a compass to draw this circle.

**Describe** the process that you used.
17. (10 points) Use a straight edge and compass to construct a rhombus with sides of length $AB$ and one angle as shown. Put one side on the line $l$.

Describe the process that you used in this problem.
Honor Code

IMPORTANT: There are multiple sections of this course. Although different versions of the exams are given in each section, there are enough similarities that sharing information about the exam could influence a student’s grade.

You are on your honor to not discuss this exam with students in other sections until after all sections have taken the test.

1. (A18 points) Using the code table below, identify the symmetry type of the following mandalas.

   ![Mandalas](image1)

   **Code for Mandalas**

<table>
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<tr>
<th>$C_n$</th>
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2. (A15 points) Using the codetable below, identify the symmetry type of these borders.

   ![Borders](image2)

   **Code for Border Patterns**

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3. (A4 points) Which of the seven types of quadrilaterals (square, rhombus, rectangle, parallelogram, kite, trapezoid and isosceles trapezoid) have double fold and cut symmetry.
4. (A8 points) For each of the following pairs of congruent figures indicate if they are related by translation, rotation, reflection or glide reflection.

(a) 
(b) 
(c) 
(d) 

5. (A9 points) In the figure below triangle ABC is first reflected across line \( l_1 \). Then the result is reflected across line \( l_2 \). The final result after two reflections is labeled as triangle \( A'B'C' \).

(\( A \) \( B \) \( C \) \( l_1 \) \( A' \) \( B' \) \( C' \) \( l_2 \))

(a) How are triangles \( ABC \) and \( A'B'C' \) related (translation, rotation, reflection or glide reflection)?

(b) Draw the three point image segments \( AA' \), \( BB' \) and \( CC' \). How many units long are these segments?

(c) What is the distance between \( l_1 \) and \( l_2 \)?

(d) What relationship do you notice between your answers in parts b) and c)? Describe.

6. (A3 points) Jane made the template design shown below. She claimed that it was a mandala of type \( D_{12} \), but one of her classmates disagreed because some of the arrows are darkened. What is the type of this mandala?

7. (A6 points) Two figures are given below which are related by a glide-reflection.

(a) Four lines \( A, B, C \) and \( D \) are given above. Indicate which line is the glide reflection line going between the two figures.

(b) Describe the process that you used to decide the glide reflection line in part a) above.

Your description:
8. (A5 points) Consider these two shapes.

(a) Give a definition of fold and cut shape so that figure a is fold and cut and figure b is not. Your definition:

(b) Give a definition of fold and cut shape so that both figure a and b are fold and cut shapes. Your definition:

9. (A10 points) For each of the following statements, decide if it is true or false.
   • If it is true, circle TRUE and you are done.
   • If it is false, circle FALSE and draw an example which shows that the statement is not always true.

(a) True? If a border has symmetry type \( \text{mm} \) then it has glide reflectional symmetry.
   Answer: TRUE or FALSE (with example)

(b) True? Borders always have translational symmetry.
   Answer: TRUE or FALSE (with example)

(c) True? Mandalas always have a line of reflection.
   Answer: TRUE or FALSE (with example)

(d) True? A mandala cannot have both reflectional and rotational symmetry.
   Answer: TRUE or FALSE (with example)
10. (11 points) Notice that the two figures given below have opposite orientations. Therefore they must be related by a glide reflection (since plane reflection does not seem to work). Using a mira, find the glide reflection line which takes one of the figures to the other.

Describe the process that you used in this problem.
CD Problem – Mira or Reflecta

Name:____________________

11. (All points) Using a mira, construct an isosceles right triangle so that the segment $AB$ is one of the legs.

*Note: Do the construction and then clearly describe the process that you used.*

*Describe:*
1. (A10 points) What is the perimeter of this figure? Express your answer with square roots and as a decimal.

   ![Figure](image)

   (a) Answer using square roots:

   (b) Answer as a decimal:

2. (A10 points) Figure out the length of the sides of these right triangles which are marked with a “?”. Show your work.

   ![Trigonometry Problems](image)

3. (A12 points) Consider the two lines pictured:

   ![Lines](image)

   First figure the slopes for the two lines:

   **Line A:**
   - Slope as ratio: __________
   - Slope as decimal: __________
   - Slope as percent: __________

   **Line B:**
   - Slope as ratio: __________
   - Slope as decimal: __________
   - Slope as percent: __________

   Next figure the lengths for the two lines:

   **Line A:**
   - Length as a square root: __________
   - Length as a decimal: __________

   **Line B:**
   - Length as a square root: __________
   - Length as a decimal: __________

4. (A6 points) By each of the three figures given below write CC for congruence condition if enough information is given to determine the figure. If not enough information is given write NOT CC.

   a) ![Triangle](image)
   b) ![Rectangle](image)
   c) ![Triangle](image)

5. (A6 points) What would be a congruence condition for a rhombus? In other words, what information is needed to determine a rhombus?
6. (A7 points) The two figures drawn below are similar. Inside of each figure write the area.

(a) What is the scale factor going from the smaller figure to the larger?

(b) What is the area factor going from the smaller figure to the larger?

(c) Describe how the area factor and the scale factor are related. Your description:

7. (A7 points) Max is planning to enlarge his square cattle pasture so that the new fence encloses two times the area of the present pasture. He wants the pasture to remain square. The original fence is 200 feet long on each side. How long should one side of the new fence be?

8. (A7 points) Jenn had a trapezoid which had area 12 and whose longest side is 5. If she scales the trapezoid so that the longest side is 15, what will the new area be?

9. (A6 points) Determine the shortest route from work to home if you must stop at both the library and the grocery store on the way. Calculate the distances for each of the two routes in the space below.

(a) Route 1

(b) Route 2

10. (A6 points) What is the slope of the hypotenuse of the triangle given below?

15

9

16
Four Step Problem

11. (A12 points)

**Official Definition:** A parallelogram is a quadrilateral in which opposite sides are equal.

**Property:** For official parallelograms, alternate interior angles for a diagonal are congruent.

*Note:* You only need to show for one diagonal. The same presentation will apply to the other diagonal as well. Alternate interior angles for the diagonal are labeled $x$ and $y$ in the figure below.

![Diagram of a parallelogram with angles labeled x and y]

**Step 1**

**Step 2**

**Step 3**

**Step 4**

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CD Problem – Straight Edge and Compass

12. (A11 points) Two sides and the included angle are given below. Using a straight edge and compass, make a triangle out of the given information. Begin by copying segment $AB$ onto line $l$ below.

First carry out your construction. Then write out a step by step description of the process that you use.

$\text{A} \quad \text{B}$

$\text{A} \quad \text{C}$

$\text{A}$

$l$
1. (A12 points) There are three copies below of a geoboard figure which has area 4. Using three different methods, show how to find this area. Be sure to name each method and show how it works.

Name of Method 1:

Name of Method 2:

Name of Method 3:

2. (A7 points) Use Pick’s formula to figure out the area of this figure. (Show your calculations!)

On the geoboard below draw a figure for which Pick’s formula does not work:

3. (A4 points) What is the measure of the angle marked with a question mark? Show your work!

4. (A4 points) A polygon has an angle sum of 1260°. How many sides does it have?
5. (A2 points) In the diagram below, the figure $ABCD$ is a parallelogram. Figure out the measure of each of the angles $a$, $b$, $c$ and $d$ (Be sure to show your work below!).

What is angle $a$?

What is angle $b$?

What is angle $c$?

What is angle $d$?

6. (A6 points) The seven types of quadrilaterals we have been working with are drawn here.

For each of the following descriptions, write down all of the names of the quadrilaterals which satisfy the description. Note: Multiple answers are possible.

(a) A quadrilateral in which there are two pairs of opposite equal sides.

(b) A quadrilateral in which there is one or more pairs of equal adjacent sides.

(c) A quadrilateral in which the diagonals are perpendicular to each other.

7. (A6 points)

(a) What is the most general shape which is both a kite and a trapezoid?

(b) What is the most general shape which is both a kite and a parallelogram?

8. (A10 points) Diagrams of quadrilaterals are given below. Decide if the information given is possible or not. If it is possible write $OK$. If there is something wrong, then write not $OK$ and explain what is the matter.

(a) $\begin{array}{c}
80^\circ \\
90^\circ \\
100^\circ \\
\text{Kite}
\end{array}$

$\text{OK or not:}$

$\text{Explain if not OK:}$

(b) $\begin{array}{c}
100^\circ \\
80^\circ \\
80^\circ \\
100^\circ \\
\text{Rhombus}
\end{array}$

$\text{OK or not:}$

$\text{Explain if not OK:}$

(c) $\begin{array}{c}
70^\circ \\
120^\circ \\
100^\circ \\
70^\circ \\
\text{Rhombus}
\end{array}$

$\text{OK or not:}$

$\text{Explain if not OK:}$
9. (A9 points) **Possible?** For each of the following statements, decide if it is possible or not.
   - If it is possible, write POSSIBLE and draw a picture.
   - If it is not possible, write NOT and give a reason.

   (a) **Possible?** A triangle which has two right angles.

   (b) **Possible?** A hexagon where all of the sides are the same length but which is not a regular hexagon.

   (c) **Possible?** A triangle where the circumcenter is on one side of the triangle.

10. (A6 points) Six different figures are given below.

   (a) Give a definition of a kite so that only the first two, $a$ and $b$, of the figures above are examples.

   (b) Give a definition of a kite so that only the first four, $a$, $b$, $c$ and $d$, of the figures above are examples.
11. (12 points) On the triangle below, use paper folding to find the center of the inscribed circle. Use a compass to draw this circle.

Describe the process that you used to find the inscribed circle.

Describe:
12. (A12 points) A segment $AB$ is drawn below. Using paper folding, construct an isosceles right triangle so that $AB$ is the hypotenuse.

First carry out your construction. Then write out a step by step description of the process that you use.